

1. Find the tangent plane to the surface $p(x, y, z) = 6$ at the point $(1, 1, 1)$ if $p(x, y, z) = x^2 + 2y^2 + 3z^2$. Write the equation of the plane as a graph over the xy plane (i.e. as a function $z = h(x, y)$).

2. Find the directional derivative of the given function at the given point P_0 and in the direction of the given vector v .

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| (a) $e^x \cos(xz)$ | $\mathbf{v} = (2, 1, -2)$ | $P_0 = (0, 0, 0)$ |
| (b) $xy + yz + xz$ | $\mathbf{v} = (0, 3, 4)$ | $P_0 = (1, 1, 2)$ |
| (c) $2xyz + x^2y$ | $\mathbf{v} = (1, 1, 1)$ | $P_0 = (2, 3, 4)$ |

3. Let $z = f(x, y) = \cos(2xy)\sin(y)$ be a surface, if you are standing on the surface at the point $(0, 0, 0)$ what direction should you move in order to increase value of z as quickly as possible.

4. Calculate the directional derivative of $f(x, y, z) = x \cos(y) \sin(z)$ at the point $a = (1, \pi/4, 5\pi/6)$ in the direction of $u = (3, 0, 1)$

5. Find a direction for which the directional derivative of the function $w(x, y, z) = y(x^2 + z^2) - z^3$ at the point $(1, 1/2, 1)$ is zero.
6. The temperature $T(x, y)$ at a point (x, y) in the plane is a differentiable function such that $\frac{\partial T}{\partial x}(3, -1) = 2$ and $\frac{\partial T}{\partial y}(3, -1) = 5$.
- (a) Find the directional derivative of T at the point $(3, -1)$ in the direction given by the unit vector $\mathbf{u} = (1/\sqrt{2}, 1/\sqrt{2})$.
- (b) A bug crawls along a level curve of the temperature function T at *speed one*. What are the possible velocity vectors \mathbf{v} for the bug at the moment it crawls through the point $(3, -1)$?
7. Compute the value of $x^2 \frac{\partial w}{\partial x} + y^2 \frac{\partial w}{\partial y}$ at the point $(1, -1)$ if $w = f\left(\frac{x+y}{xy}\right)$ and $f'(0) = -2$.