1. Match the following using integral theorems we learned this semester. Be sure to check the hypotheses of the theorem, and if possible, compute the integral.

(a) $\displaystyle\iint_{x^2+y^2+z^2=1} \left(x^3/3, z(zy+x), y^2z\right)\, ds$

(b) $\displaystyle\int_0^1\int_0^{2\pi}\int_0^{\pi} \rho^4\sin(\varphi)\, d\varphi\, d\theta\, d\rho$

(c) $\displaystyle\int_{c(t)} \left(\frac{y}{2}, \frac{-x}{2}\right) ds$ With $c(t) = \begin{pmatrix} t^3-t \\ t^2 \end{pmatrix}$, $-1 \le t \le 1$.

(d) $\displaystyle\iint_{\frac{x^2}{4}+\frac{y^2}{16}\le 1} xy\, dA$

(e) $\displaystyle\int_{c(t)} \begin{pmatrix} y+ze^{xy} \\ x+2z\cos(yz)\sin(yz) \\ 2y\cos(yz)\sin(yz)+xe^{xy} \end{pmatrix} dS$

With $c(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \\ 0 \end{pmatrix}$, $0 \le t \le \pi$.

(f) $\displaystyle\int_{c(t)} \nabla\times F(x,y,z)\, dS$ With $\Phi(s,t) = \begin{pmatrix} s\cos(t) \\ s\sin(t) \\ 2s \end{pmatrix}$, $0 \le t \le 2\pi$ and $F(x,y,z) = \left(x^4z, x^2+y^2-\frac{z}{2}, \tan(x^2y^3z)\right)$.

(α) $\displaystyle\int_{-1}^{0}\int_{-\sqrt{x^3-x}}^{\sqrt{x^3-x}} 1\, dA$

(β) $f(-1,0,0) - f(1,0,0)$ With $f = xy + \cos(yz)^2 + e^{xz}$

(γ) $\displaystyle\int_{c(t)} F(x,y,z)\, dS$ With $c(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \\ 2 \end{pmatrix}$, $0 \le t \le 2\pi$ and $F(x,y,z) = \left(x^4z, x^2+y^2-\frac{z}{2}, \tan(x^2y^3z)\right)$.

(δ) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{1-x^2-y^2}^{1-x^2-y^2} x^2+y^2+z^2\, dz\, dy\, dx$

(ϵ) $\displaystyle\int_0^{2\pi}\int_0^{16} 32r^3\sin(2\theta)\, dr\, d\theta$

(ζ) $\displaystyle\int_{c(t)} \begin{pmatrix} y+ze^{xy} \\ x+2z\cos(yz)\sin(yz) \\ 2y\cos(yz)\sin(yz)+xe^{xy} \end{pmatrix} dS$

With $c(t) = \begin{pmatrix} 1-2t \\ 0 \\ 0 \end{pmatrix}$, $0 \le t \le 1$.

2. Your book defines the first and second order Taylor polynomials on page 196. Using more compact notation, we can write the Taylor Formula for the function f about the point x_0 as

$$p_2(x) = f(x_0) + Df(x_0)(\mathbf{x} - x_0) + (\mathbf{x} - x_0)^t Hf(x_0)(\mathbf{x} - x_0)$$

(a) Write out Taylor's Formula explicitly for scalar valued functions of one, two, and three variables.

(b) Using the function $f(x,y) = \sin(xy)$, compute the Hessian matrix of f at the point $\mathbf{a} = (0,0)$.

(c) Compute the degree 2 Taylor Polynomial of $f(x,y)$ at the point $\mathbf{a} = (0,0)$

3. Find the second order Taylor expansion about the point $(0,0)$ of the function

$$f(x,y) = e^{xy}$$

4. Find and classify all critical points of the function

$$f(x,y) = x^3 + x^2y + y^2 + xy + x + 1$$

5. There has been an oil spill in a triangular ocean. The ocean has vertices at the points $(1,0), (0,1)$, and $(0,0)$. The plan to disperse the oil calls for installing an oil vacuum rig (which sucks up oil from the surface of the ocean) at the point (x,y). The amount of oil removed by the rig is proportional to the product of the distances from the three edges. Where should the oil vacuum be placed to most effectively remove the oil?

   Remember that the distance from (x,y) to the line $ax + by = c$ is given by $d = |ax + bc + c| / \sqrt{a^2 + b^2}$. This ocean is populated entirely by baby seals, singing disneyfish, and the rare fluffy northern penguin, failure is not an option.