Solution to FM 5002 Homework 1

Problem 1-1

$$\begin{aligned} E[X] &= 4 \cdot Pr[X=4] + 10 \cdot Pr[X=10] = 4 \cdot 0.65 + 10 \cdot 0.35 = 2.6 + 3.5 = 6.1 \\ Var[X] &= E[X^2] - (E[X])^2 = 4^2 \cdot Pr[X=4] + 10^2 \cdot Pr[X=10] - (6.1)^2 = 8.19 \\ SD[X] &= \sqrt{Var[X]} = 2.861818 \end{aligned}$$

Problem 1-2

1-2(a)

$$Pr[(Y=3)|(X=4)] = \frac{Pr[(Y=3)\&(X=4)]}{Pr[X=4]} = \frac{0.2}{0.65} = 4/13 = 0.3076923.$$

1-2(b)

$$\begin{aligned} E[Y|(X=4)] &= 3 \cdot Pr[(Y=3)|(X=4)] + 7 \cdot Pr[(Y=7)|(X=4)] \\ &= 3 \cdot Pr[(Y=3)\&(X=4)]/Pr[X=4] + 7 \cdot Pr[(Y=7)\&(X=4)]/Pr[X=4] \\ &= 75/13 = 5.769231. \end{aligned}$$

Problem 1-3

$$\begin{aligned} Pr[B|A] &= \frac{Pr[B\&A]}{Pr[A]} \\ &= \frac{Pr[A|B]Pr[B]}{Pr[A]} \\ &= \frac{0.3 \cdot 0.8}{0.4} \\ &= 0.6. \end{aligned}$$

Problem 1-4

$$Odds[A|(B\&C)] = \frac{Pr[A|(B\&C)]}{1 - Pr[A|(B\&C)]}$$

And

$$\begin{aligned} Pr[A|(B\&C)] &= \frac{Pr[A\&B\&C]}{Pr[B\&C]} \\ &= \frac{Pr[C|A\&B]Pr[A\&B]}{Pr[B\&C]} \\ &= \frac{Pr[C|A\&B] \cdot Pr[B|A] \cdot Pr[A]}{Pr[B\&C]} \end{aligned}$$

Since

$$\begin{aligned}
Odds[A] = 3/2 &\Rightarrow \frac{Pr[A]}{1 - Pr[A]} = 3/2 \Rightarrow Pr[A] = 0.6 \\
Pr[B] &= Pr[B|A] \cdot Pr[A] + Pr[B|A^c] \cdot Pr[A^c] = 0.4 * 0.6 + 0.4 * (1 - 0.6) = 0.4. \\
Pr[B\&A] &= Pr[B|A] \cdot Pr[A] = 0.24. \\
Pr[B\&A^c] &= Pr[B|A^c] \cdot Pr[A^c] = 0.16. \\
Pr[B\&C] &= Pr[C\&B\&A] + Pr[C\&B\&A^c] \\
&= Pr[C|B\&A] \cdot Pr[B\&A] + Pr[C|B\&A^c] \cdot Pr[B\&A^c] \\
&= 0.2 * 0.24 + 0.1 * 0.16 = 0.064. \\
Pr[A|(B\&C)] &= \frac{0.2 * 0.4 * 0.6}{0.064} = 0.048/0.064 = 0.75. \\
&\Rightarrow Odds[A|(B\&C)] = 0.75/(1 - 0.75) = 3.
\end{aligned}$$

Problem 1-5

Problem (a), (b), (c), (d) are on the attached paper.

e

By definition, a **Partition** of a PCRV v is $\{V^{-1}|y \in V[0,1]\}$, then

$$\wp = \{[0, 0.4), [0.4, 1]\}.$$

f

$$\Re = \{[0, 0.5), [0.5, 1]\}.$$

g

Since the partition of X belongs to $\wp$, then X is $\wp$-measurable.

h

Since $Y = -2$ in the subset $[0.4, 0.5)$, while in the subset $[0.5, 1]$, $Y = -1$, then Y is not constant in the subset $[0.4, 1]$ of partition $\wp$. Y is not $\wp$-measurable.

i

Since the partition of $E[Y|X]$ is $\{[0, 0.4), [0.4, 1]\} = \wp$, then $E[Y|X]$ is $\wp$-measurable.

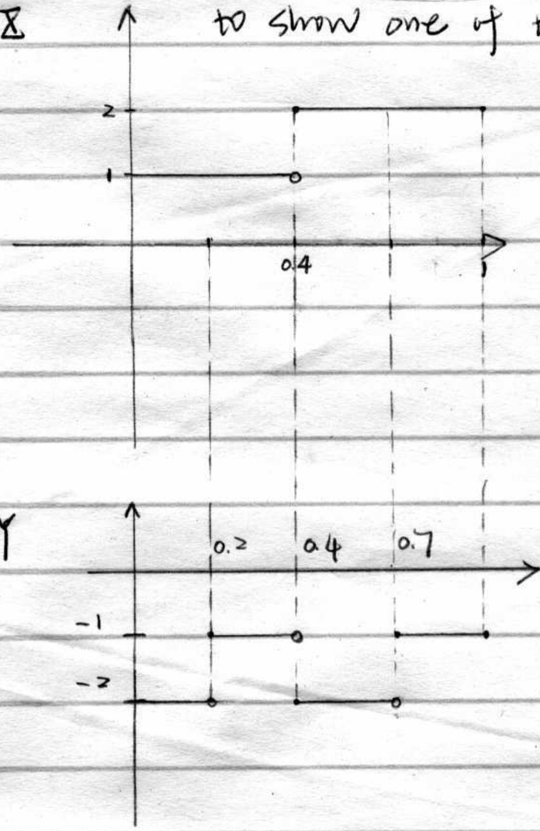
j

Since X and Y are different, then $E[XY] = E[X]E[Y]$, and

$$covar[X, Y] = E[XY] - E[X]E[Y] = 0.$$

There can be many PLPVs satisfy the conditions, you just need to show one of them.

(a) X



(b) Y

By independence, we should have

$Pr[X=1, Y=-1] = 0.4 \cdot 0.5 = 0.2$

$Pr[X=1, Y=-2] = 0.4 \cdot 0.5 = 0.2$

$Pr[X=2, Y=-1] = 0.6 \cdot 0.5 = 0.3$

$Pr[X=2, Y=-2] = 0.6 \cdot 0.5 = 0.3$

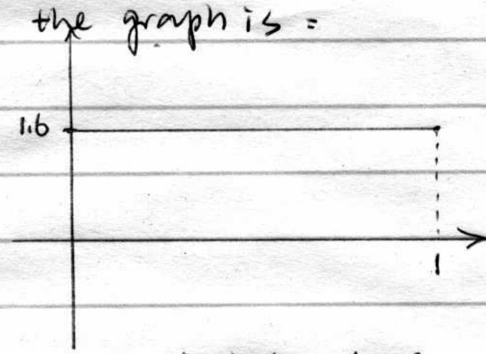
(c) $E[X \mid Y=-1] = 1 \cdot Pr[X=1 \mid Y=-1] + 2 Pr[X=2 \mid Y=-1]$

(By independence) $= 1 \cdot Pr[X=1] + 2 \cdot Pr[X=2]$

$= 1 \cdot 0.4 + 2 \cdot 0.6 = 1.6$

and $E[X \mid Y=-2] = 1.6$

then the graph is:



(d) $E[Y \mid X] \overset{\text{By independence}}{=} -1 \cdot 0.5 - 2 \cdot 0.5 = -1.5$