04/09 -1

1. By Product Rule:

$$\frac{\partial}{\partial x}[f \cdot g] = \frac{\partial f}{\partial x} g + \frac{\partial g}{\partial x} f$$

$$\frac{\partial}{\partial x}\left[e^{x^2y}(\sin(x+y))\right] = e^{x^2y}\left[2xy\sin(x+y) + \cos(x+y)\right]$$

2. 1

3. $$\frac{\partial}{\partial y}\left[e^{x^2y}\sin(x+y)\right] = e^{x^2y}\left[x^2\sin(x+y) + \cos(x+y)\right]$$

4. 1.

5. gradient $\nabla f =$

$$\left(e^{x^2y}[\sin(x+y)\cdot 2xy + \cos(x+y)],\ e^{x^2y}[x^2\sin(x+y) + \cos(x+y)]\right)$$

6. $\nabla f(0,0) = (1, 1)$

7. 1-jet @ $(x,y) = (0,0)$

$f(0,0) = 0$, $\frac{\partial}{\partial x} f(0,0) = 1$, $\frac{\partial}{\partial y} f(0,0) = 1$

8. $p(x,y) = 0 + 1 \cdot x + 1 \cdot y = x + y$.

9. $$\frac{\partial^2}{\partial x^2} e^{x^2y}\sin(x+y) = e^{x^2y}\left[(4x^2y^2 + 2y - 1)\sin(x+y) + 4xy\cos(x+y)\right]$$

10. $x=0, y=0. \Rightarrow \frac{\partial^2}{\partial x^2} e^{x^2y}\sin(x+y) = 0$

11. $$\frac{\partial^2}{\partial x \partial y}\left[e^{x^2y}\sin(x+y)\right] = e^{x^2y}\left[(2x^3y + 2x - 1)\sin(x+y) + (x^2+2xy)\cos(x+y)\right]$$

12. 0

13. $\frac{\partial^2}{\partial y^2}\left[e^{x^2y}(\sin(x+y))\right] = e^{x^2y}\left[(x^4-1)\sin(x+y) + 2x^2\cos(x+y)\right]$

14. $\frac{\partial^2}{\partial y^2}\left[e^{x^2y}\sin(x+y)\right]\Big|_{\substack{x=0\\y=0}} = 0$

15. Hessian is

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x\partial y} \\ \frac{\partial^2 f}{\partial x\partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = e^{x^2y}\begin{bmatrix} (4x^2y^2+2y-1)\sin(x+y)+4xy\cos(x+y) & (2x^3y+2x-1)\sin(x+y) + (x^2+2xy)\cos(x+y) \\ (2x^3y+2x-1)\sin(x+y) + (x^2+2xy)\cos(x+y) & (x^4-1)\sin(x+y) + 2x^2\cos(x+y) \end{bmatrix}$$

16 $$\begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x\partial y} \\ \frac{\partial^2 f}{\partial x\partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}\Bigg|_{x=0,\, y=0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

17. 2-jet of $e^{x^2y}\sin(x+y)$ w.r.t. (x,y) @ $(x,y)=(0,0)$

$f(0,0)=0$, $\frac{\partial}{\partial x}f(0,0)=1$, $\frac{\partial}{\partial y}f(0,0)=1$,

$\frac{\partial^2}{\partial x^2}f(0,0)=0$, $\frac{\partial^2}{\partial x\partial y}f(0,0)=0$, $\frac{\partial^2}{\partial y^2}f(0,0)=0$,

18. 2nd order Macl. approximation is

$p(x,y) = x+y$.

19. The question is how can we distribute 10 derivatives to 7 variables.

Then for 10th derivative term, the number is $\binom{10+7-1}{7-1} = \binom{16}{6}$

for kth derivative, the number is $\binom{k+7-1}{7-1} = \binom{k+6}{6}$ ...

Therefore, consider all derivative of order k from 0 to 10, the number is $\sum_{k=0}^{10}\binom{k+6}{6} = \sum_{k=0}^{10} C^6_{k+6}$.