Solution to FM 5022 Homework 2

Problem 19.3

By EWMA model (*Formula 19.7*),

$$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) u_{n-1}^2.$$

In our case, $\sigma_{n-1} = 1.5\% = 0.015$ and $u_{n-1} = (30.5 - 30)/30 = 0.5/30 = 1/60$. Therefore

$$\sigma_n = \sqrt{0.94 \times 0.015^2 + (1 - 0.94) \times 1/60^2} = 0.015105 = 1.15105\%.$$

Problem 19.4

By EWMA modle,

$$\sigma_n^2 = \lambda \sigma_{n-1}^2 + (1-\lambda) u_{n-1}^2.$$

Reducing λ would put more weight on the recent observation (u_{n-1}) and less weight on the older data (σ_{n-1}).

Problem 19.6

GARCH(1,1) Model is

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2,$$

where $\gamma = 1 - \alpha - \beta$. Setting $\omega = \gamma V_L$, we have

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2.$$

Therefore, increasing ω will put more weight on the long-run average variance rate.
Increasing α would put more weight on the most recent data item, reduces the weight given to the long-run average variance rate (since $V_L = \omega/\gamma = \frac{\omega}{1-\alpha-\beta}$).
Increasing β would put more weight on the previous variance estimate, reduces the weight given to the long-run average variance rate.

Problem 19.8

$u_{n-1} = (1060 - 1040)/1040 = 20/1040 = 1/52 \Rightarrow$

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 = 0.000002 + 0.06 \times 0.01923^2 + 0.92 \times 0.01^2 = 0.0001162.$$

$$\Rightarrow \sigma_n = \sqrt{0.0001162} = 0.01078.$$

Problem 19.11

The GARCH(1,1) gives

$cov_n = \gamma V_L + \alpha u_{n-1} v_{n-1} + \beta cov_{n-1} = 0.000001 + 0.04 \times 0.0333 \times 0.02 + 0.94 \times 0.00006 = 0.0000841.$

And also

$$\sigma_{u,n} = 0.01189 = 1.189\%.$$

$$\sigma_{v,n} = 0.01242 = 1.242\%.$$

Therefore,

$$corr = 0.0000841/(0.01189 \times 0.01242) = 0.569.$$

Problem 19.14

Consider the variable u_{n-1}^2 is σ_{n-1}^2 and variance $2\sigma_{n-1}^2$. Assume u_i is generated by a Wiener process dz, then

$$u_{n-1}^2 = \sigma_{n-1}^2 + \sqrt{2}\sigma_{n-1}^2\epsilon.$$

Then since

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2,$$

then

$$\sigma_n^2 - \sigma_{n-1}^2 \quad = \quad \omega + \alpha u_{n-1}^2 + (\beta - 1)\sigma_{n-1}^2$$

Let $\Delta V = \sigma_n^2 - \sigma_{n-1}^2$, and $V = \sigma_{n-1}^2$, $a = 1 - \alpha - \beta$, $aV_L = \omega$, and $\xi = \alpha\sqrt{2}$, so that

$$\Delta V = a(V_L - V) + \xi\epsilon V.$$

When it's measured in days, $\Delta t = 1$,

$$\Delta V = a(V_L - V)\Delta t + \xi\epsilon V\sqrt{\Delta t}.$$

When the time is measured in years, $\Delta t = 1/252$ then

$$\Delta V = a(V_L - V)252\Delta t + \xi\epsilon V\sqrt{252\Delta t}.$$