Solution to FM 5022 Homework 8

Problem 26.1

$$L \times \delta_k \times (R_k - R_K) = 20 \times 0.25 \times (12\% - 10\%) = \$100,000$$

would be paid out 3 month later.

Problem 26.2

A swap option (or swapoption) is an option to enter into an interest rate swap at a certain time in teh future with a certain fixed rate being used.

An interest rate swap can be regarded as the exchange of a fixed-rate bond for a floating-rate bond. A swaption is therefore an option to exchange a fixed-rate bond for a floating-rate bond. The floating-rate bond will be worth its face value at the beginning of the life of the swap. The swaption is therefore an option on a fixed-rate bond with the strike price equal to the fae value of the bond.

Problem 26.3

By Black's model, $F_0 = (126 - 10)e^{0.1\times1} = 127.09, K = 110, P(0,T) = e^{-0.1\times1}, \sigma_B = 0.08$, and $T = 1.0$.

$$\begin{aligned} d_1 &= \frac{\ln(F_0/K) + \sigma_B^2/2}{\sigma} = 1.8456; \\ d_2 &= \frac{\ln(F_0/K) - \sigma_B^2/2}{\sigma} = 1.7656; \end{aligned}$$

The value of the put option is

$$P(0,T)[KN(-d_2) - F_0N(-d_1)] = 0.12.$$

Problem 26.5

By formula (26.13), $L = 1000, \delta_k = 0.25, F_k = 0.12, R_K = 0.13, r = 0.115, \sigma_k = 0.12, t_k = 1.25, P(0, t_{k+1}) = 0.8416$.

$$L\delta_k = 250;$$

$$\begin{aligned} d_1 &= \frac{\ln(F_k/R_K) + \sigma_k^2 \times t_k/2}{\sigma\sqrt{t_k}} = -0.5925; \\ d_2 &= d_1 - \sigma^2\sqrt{t_k} = -0.6637; \end{aligned}$$

The value of the option is

$$L\delta_k \times [F_kN(d_1) - R_KN(d_2)] = 0.59.$$

Problem 26.7

The PV of the principal in the four year bond is $100e^{-4\times0.1} = 67.032$, the PV of the coupon is, therefore, $102 - 67.032 = 34.968$. This means that the forward price of the five-year bond is

$$(105 - 34.968)e^{4\times0.1} = 104.475.$$

The parameters in Black's model are therefore $F_0 = 104.475, K = 100, r = 0.1, T = 4$, and $\sigma = 0.02$.

$$\begin{aligned} d_1 &= \frac{\ln 1.04475 + 0.5 \times 0.02^2 \times 4}{0.02\sqrt{4}} = 1.1144 \\ d_2 &= d_1 - 0.02\sqrt{4} = 1.0744. \end{aligned}$$

The price of the European call is

$$e^{-0.1\times 4}[104.475N(1.1144) - 100N(1.0744)] = 3.19.$$

Problem 26.14

By formula (26.15)

$$A = \frac{1}{m}\sum_{i=1}^{mn} P(0,T_i) = \frac{1}{(1+5\%)^5} + \frac{1}{(1+5\%)^6} + \frac{1}{(1+5\%)^7} = 2.2404$$

$$\begin{aligned} d_1 &= \frac{\ln(S_0/S_k) + \sigma^2 T/2}{\sigma\sqrt{T}} = 0.2; \\ d_2 &= d_1 - \sigma\sqrt{T} = -0.2; \\ C &= LA[S_0N(d_1) - S_kN(d_2)] = 0.1776. \end{aligned}$$

Problem 26.18

We consider two portfolios:
The first: consisting of the swap option to receive s_K;
The second: consisting of the swap option to pay s_K and the forward swap to receive s_K.
Suppose that the actual swap rate at the maturity of the options is greater than s_K. The first swaption will not exercised. The second swaption will be exercised, then is neutralized by the forward swap.
Suppose that the actual swap rate at the maturity of the options is less than s_K. The first swaption will be exercised. The second swaption will not be exercised. Forward swap always exercised. Both portfolios then are equivalent to a sap where s_K is is received and floating is paid. In all states the two portfolios worth the same at time T_1. They must worth the same today.
When s_K equals the current forward swap option to receive fixed when the fixed rate in the swap option is the forward swap rate.