PROBLEMS BASED ON THE SPRING 2003 MATHEMATICS STANDARDS AND BENCHMARKS FOR GRADE 3 WITH COMMENTS

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The purpose of this link from my web-site is to identify a selection of problems aligned with the Minnesota mathematics standards and benchmarks for Grade 3 as adopted in Spring 2003. My focus consists of the standards and benchmarks themselves; the problems here serve to illuminate them. The benchmarks and standards that are particularly relevant for a particular problem are identified in the left-hand margins; for instance, 3-II.B.1 indicates the Grade-3 benchmark II.B.1 and 3-II.B refers to the corresponding standard. In another sense, the focus is the suitability of problems for the Minnesota Comprehensive Assessments (know as MCA's), but in saying this I want to emphasize that the opinions are mine alone, formed without consultation with Minnesota Department of Education. This particular link also includes a variety of comments about the problems. A problem list without this commentary is on another link.

I was one of approximately 40 members of the mathematics subcommittee of the Academic Standards Committee, formed by the Minnesota Commissioner of Education in February 2003. I strongly support the mathematics standards and benchmarks resulting from the work of that committee and which, on the basis of a law passed by the Legislature and signed by the Governor, became official in Spring 2003. Although there is no guarantee that this web-site item reflects the thinking within the Department of Education, I have tried very hard to reflect the standards and benchmarks accurately, taking care not to bend them in the direction of my individual views. [Even though I strongly support the standards and benchmarks document, there are places where I would have preferred the document to be a bit different, and I suspect that the same is true (but not for the same places) of every member of the mathematics subcommittee.]

Anticipating that I might want to modify this document from time to time, I have refrained from labeling the problems with numerals and am planning to change the date at the top any time I make additions or changes.

Since the standards are cumulative, all the K-3 benchmarks are relevant for the Grade-3 MCA. It seems to me that it is desirable for K-3 teachers to examine all the K-3 benchmarks, and in general for teachers to read the standards for a couple grades on either side of the grade they are teaching.

Even though I view all the problems below as consistent with the Grade-3 standards and benchmarks, the range of difficulty represented by them is wide. I have chosen the adjectives 'standard', 'substantial' and 'challenging' for the problems. The challenging problems are those that, in my opinion, require significantly more than mastery of individual benchmarks. A substantial problem is one that has a feature of richness beyond what would be expected in a standard problem and yet which comes short of being the challenge that a challenging problem would represent. Among the standard problems I use two labels: standard-e, and standard-h indicating a distinction between easier and harder standard problems. This assignment of level of difficulty will follow each problem. But these personal opinions of mine are of secondary importance compared to the central issue of alignment of problems with benchmarks. I want to emphasize that the challenging and substantial problems are aligned with the benchmarks; it is not that they are on topics that go beyond the standards. [For an analogy, I mention a long-standing phenomenon with some standardized tests—a seventh grader might be told that he or she has preformed at, say, the tenth-grade level. This does not mean that the seventh grader knows tenth-grade mathematics, but rather that he or she does as well on seventh-grade material as would an average tenth grader.]

The variety of different problems that are consistent with the standards and benchmarks is very large—that is the power of mathematics; a manageable number of basic principles and techniques enables one to handle a myriad of different situations. So, of course, the problem list that follows cannot be viewed as comprehensive.

For each problem in which students are to place the correct digits in boxes, I have included three boxes. If the answer requires only two digits or one, it is the left-hand box or boxes which should be left blank. [If the Grade-3 MCA were, in fact, to include such problems it would be important that students become familiar with the instructions some days in advance of the test.]

It is clear from the benchmarks that the Grade-3 MCA should consist mostly of problems for which a calculator should not be permitted. The small latter portion of the problem list below is there in case the state decides that there should also be a 'calculator permitted' portion of the Grade-3 MCA; the place where this latter portion begins is clearly identified with an introductory sentence.

I want to again emphasize: Although the standards and the benchmarks accompanying them constitute an official document of the state of Minnesota, all the judgments about alignment of problems with the benchmarks and standards are mine; neither do they have any official standing nor have they been obtained in consultation with the Minnesota Department of Education. Also, they have not been reviewed by the University of Minnesota where I am a faculty member and, of course, they do not represent any official view of that institution.

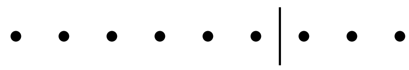
3-III.B.1 What number belongs in the blank?

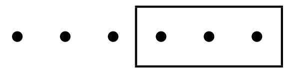
$$\underline{\hspace{1.5em}} + 5 = 9$$

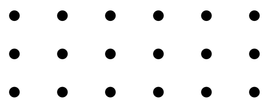
(a) 4
(b) 5
(c) 9
(d) 14

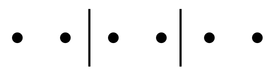
Difficulty: standard-e.

3-II.B.6 Which of the following pictures is a picture of 6×3?

(a) 

(b) 

(c) 

(d) 

Difficulty: standard-e. A more accurate presentation of the problem might be: Which of the following pictures best represents 6×3? However, the word ‘represents’ might be troublesome for third graders, so that is why I have chosen to write the problem as I have.

3-II.A.1 Which number is four thousand, two hundred seven?

(a) 427
(b) 4027
(c) 4207
(d) 4270

Difficulty: standard-e.

3-V.A.1 How many lines of symmetry does a square have?

(a) 0
(b) 1
(c) 2
(d) 4

Difficulty: standard-h.

3-III.B.2
3-I.1

When calculating $8 + 5 + 7$ you can first add any two of the numbers first and then add the third number. When doing this, which of the following would not happen?

(a) $12 + 8$
(b) $13 + 7$
(c) $14 + 6$
(d) $15 + 5$

Difficulty: standard-h. At this grade level, the easy path of noticing that 6 is not an original summand should not be expected. Also, the word 'not' near the end of the problem can be troublesome.

3-V.C
3-I.1

Which statement is correct?

(a) 3 feet equals 1 yard.
(b) 100 meters equals 1 centimeter.
(c) 60 hours equals 1 minute.
(d) 4 dollars equals 1 quarter.

Difficulty: standard-h. I wonder if the word 'statement' should be omitted from the problem. Without it, the problem still seems to be clear, and 'statement' is a word that some students might not know.

3-II.B.2
3-I.2
3-I.5

Deborah had 133 pieces of candy, but she gave 84 pieces away. Which of the following equals the number of pieces of candy she still has?

(a) $133 - 84$
(b) $133 - 49$
(c) $133 + 49$
(d) $133 + 84$

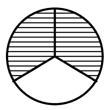
Difficulty: standard-h. The benchmark 3-I.2 is involved even though there is no irrelevant information in the problem; there is irrelevant information in the detractors—the result 49 obtained by subtracting 84 from 133 is irrelevant. By comparing the standards 3-II.B and 4-II.B along with their accompanying benchmarks one sees that actually asking students to carry out the calculation $133 - 84 = 49$ without a calculator would be ok for the Grade-4 MCA but not the Grade-3 MCA. This is a good place to discuss school districts that have different approaches to teaching subtraction. Approach (i): Teach students how to do subtraction simultaneously with teaching them how to recognize when it is useful for solving real-world problems. There is nothing in the standards that prevents a school district from teaching and testing subtraction skills in Grade 3, even though the actual subtraction algorithm should not be needed for a Grade-3 MCA—and in any system of instruction there will be some overlap between Grade 3 and Grade 4. Approach (ii): It is important that students learn what subtraction means before they are asked many repetitive type exercises designed to enhance their subtraction skills; often students leap to a calculation before thinking about which calculation is really needed, and teaching them about the meaning of subtraction before doing lots of subtractions can help them learn to think before leaping. The standards accommodate this view provided that the school district does not delay the skill practice beyond the end of April in Grade 4. Approach (iii): Skill at subtracting very long numerals is important because it teaches the recursive nature of algorithms. A school district taking this view can test for it in their classes, but nowhere do the standards and benchmarks ask for skill at subtracting one seven-digit number from another without a calculator. Approach (iv): Understanding subtraction should be the focus. Calculators are always available for doing a subtraction, so students should not have to learn how to do it by hand, especially not how to borrow (or other equivalent methodology). The standards and benchmarks explicitly disagree with this view, and so for students to do well on aligned MCA's such a school district will need to make modifications. I have merely taken subtraction as an example; many other topics would have served equally well. The purpose of this rather long discussion was not to get into a philosophical discussion of local versus state control, but rather to give an indication of the actual nature of that balance under the new standards and benchmarks.

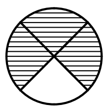
3-V.C.3 Which of the following is correct?

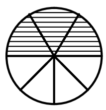
(a) 12 feet equals 1 yard
(b) 12 yards equals 1 foot
(c) 12 feet equals 1 inch
(d) 12 inches equals 1 foot

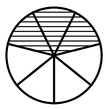
Difficulty: standard-h. One might regard standard-e to be an appropriate difficulty rating, but I have given the rating standard-h because of the reading discipline required in order to give a correct response.

3-II.A.4
3-IV.A.1 Which of the following pictures shows the fraction 3/4?

(a) 

(b) 

(c) 

(d) 

Difficulty: standard-h. The correct answer is (b), but one could make a case for (d) if one were thinking of a setting involving proportions. However, at Grade 3 I think the problem is unambiguous. I did consider replacing the word 'shows' with the phrase 'best represents', but I decided against doing so since inclusion of the phrase 'best represents' might place the problem beyond the reading level of some third graders.

3-V.B.1
3-I.6

Which is always true about right triangles?

(a) Two sides are equal.
(b) There are no acute angles.
(c) There is exactly one acute angle.
(d) The triangle can be put together with a copy of itself in order to form a rectangle.

Difficulty: substantial. Benchmark 3-I.6 is relevant because the student can help herself or himself by sketching a couple right triangles with somewhat different appearances.

2-III.B.1
3-II.A.1

Find $539 + 0$.

Difficulty: standard-e.

3-II.B.3

If you find $5,978 + 7,836$ and then subtract $5,978$ from that sum, what will you finally get?

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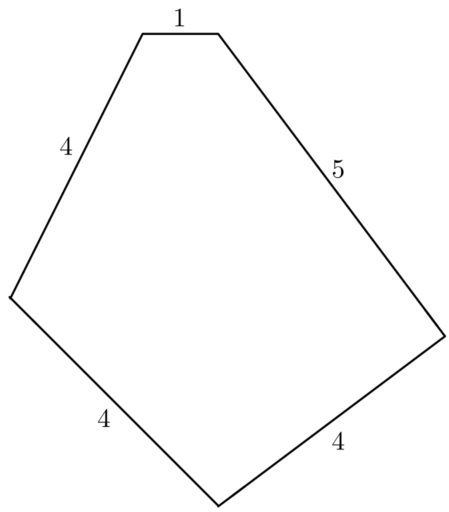
Difficulty: standard-h. On purpose the arithmetic involved has been made difficult since the purpose of the problem is to check if students understand that subtraction reverses addition.

3-V.C.2 Find the perimeter of the polygon shown below.



Difficulty: standard-h. An issue: Should the word ‘polygon’ be replaced by ‘pentagon’ in the statement of the problem—or should both these words be avoided by making direct reference to the picture?

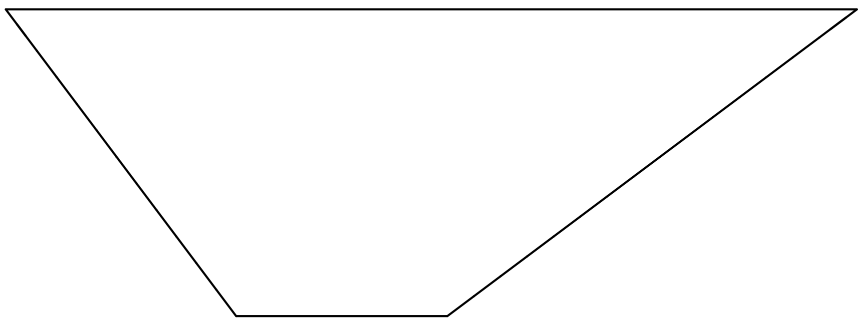
3-V.C.5
2-V.C.2

How many minutes is it from 11:48am to 12:27pm of the same day?

Difficulty: challenging. Knowing how to add and subtract multi-digit numbers in general without a calculator is only identified as a skill in Grade 4. However, the tone of 3-II.B and 3-V.C for indicates that limited skill of this sort is to be expected in Grade 3. The am-pm-same day aspects of the problem makes the reading difficult at the Grade-3 level even though the am-pm issue is entailed in a Grade-2 benchmark.

3-V.B.2
3-I.6

Show how the following shape is composed of two triangles and one rectangle. Label the parts of your picture.



Difficulty: substantial. I am not feeling completely comfortable with my sentence "Label the parts of your picture." I want to say 'Clearly communicate your answer', but I want to do so in a way that is understandable to a third grader. The problem does not refer to the figure as a trapezoid—that is fine for Grade 3. Were this a high school or possibly a middle school problem, something would have to be said to assure the student that the top and bottom edges are actually parallel rather than just close to parallel. But at such higher grade levels one can describe the problem more precisely without creating an unreasonable hurdle for students.

If it can be managed, say with a voice-activated machine, I can envision part of the **MCA's for Grade 3 as being oral.** A problem might ask for a student to do as many of the simple calculations as he or she can in 2 minutes, with scoring based on the total number tried, total number correct, and total number incorrect. If such testing cannot be managed, then several problems similar to those below should appear on the Grade-3 MCA's, preferably as problems where the student has to fill in boxes with digits of the correct answers.

3-II.B.4
3-II.B.5

$7+9$	$6+2$	$6-3$	$13-9$
$6+8$	$8-0$	$7+3$	$12-6$
$0+4$	$9-6$	$0+7$	$5-5$
$12-3$	$9+6$	$1+7$	$7-7$
$7+1$	$2+8$	$5+7$	$11-4$
$18-9$	$14-7$	$3+3$	$1+1$
$7+2$	$4+8$	$6-1$	$2+2$
$13-6$	$5+5$	$4+6$	$7-6$
$2+0$	$3-0$	$0+9$	$16-8$
$3+8$	$9-2$	$9-8$	$9+7$
$9+3$	$9-4$	$7-4$	$12-7$
$8-4$	$6+6$	$6-6$	$2+5$
$2+7$	$7+6$	$11-9$	$6+5$
$9+4$	$3-2$	$5+3$	$9+1$
$17-9$	$6+4$	$6-4$	$5+5$

FOR GRADE 3, WITH COMMENTS

I include below a problem which would be appropriate for a **calculator portion of the Grade-3 MCA** if indeed there is such a portion.

3-II.B.1
3-II.B.2
3-I.5
3-I.2

Alphonso attended school 166 days as a first grader and 165 days as a second grader. How many days does he have to attend as a third grader in order that his total attendance for the three grades equal 500 days?

Difficulty: substantial. Remembering that we should not expect neat shortcuts from a third grader, this problem requires careful reading and a step-by-step approach.

The views and opinions expressed in this link are strictly those of Bert Fristedt. The contents have been neither reviewed nor approved by the University of Minnesota.