PROBLEMS BASED ON THE SPRING 2003
MATHEMATICS STANDARDS AND BENCHMARKS

FOR GRADE 4
WITH COMMENTS

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The purpose of this link from my web-site is to identify a selection of problems aligned with the Minnesota mathematics standards and benchmarks for Grade 4 as adopted in Spring 2003. My focus consists of the standards and benchmarks themselves; the problems here serve to illuminate them. The benchmarks and standards that are particularly relevant for a particular problem are identified in the left-hand margins; for instance, 4-V.C.3 indicates the Grade-4 benchmark V.C.3 and 4-V.C refers to the corresponding standard. In another sense, the focus is the suitability of problems for the Minnesota Comprehensive Assessments (know as MCA's), but in saying this I want to emphasize that the opinions are mine alone, formed without consultation with Minnesota Department of Education. This particular link also includes a variety of comments about the problems. A problem list without this commentary is on another link.

I was one of approximately 40 members of the mathematics subcommittee of the Academic Standards Committee, formed by the Minnesota Commissioner of Education in February 2003. I strongly support the mathematics standards and benchmarks resulting from the work of that committee and which, on the basis of a law passed by the Legislature and signed by the Governor, became official in Spring 2003. Although there is no guarantee that this web-site item reflects the thinking within the Department of Education, I have tried very hard to reflect the standards and benchmarks accurately, taking care not to bend them in the direction of my individual views. [Even though I strongly support the standards and benchmarks document, there are places where I would have preferred the document to be a bit different, and I suspect that the same is true (but not for the same places) of every member of the mathematics subcommittee.]

Anticipating that I might want to modify this document from time to time, I have refrained from labeling the problems with numerals and am planning to change the date at the top any time I make additions or changes.

Since the standards are cumulative, all the K-4 benchmarks are relevant for the Grade-4 MCA. It seems to me that it is desirable for Grade-4 teachers to examine all the K-4 benchmarks giving special attention to those for grades 2-4, and in general for teachers to read the standards for a couple grades on either side of the grade they are teaching.

Even though I view all the problems below as consistent with the Grade-4 standards and benchmarks, the range of difficulty represented by them is wide. I have chosen the adjectives 'standard', 'substantial' and 'challenging' for the problems. The challenging problems are those that, in my opinion require sig-

nificantly more than mastery of individual benchmarks. A substantial problem is one that has a feature of richness beyond what would be expected in a standard problem and yet which comes short of being the challenge that a challenging problem would represent. Among the standard problems I use two labels: standard-e, and standard-h indicating a distinction between easier and harder standard problems. This assignment of level of difficulty will follow each problem. But these personal opinions of mine are of secondary importance compared to the central issue of alignment of problems with benchmarks. I want to emphasize that the challenging and substantial problems are aligned with the benchmarks; it is not that they are on topics that go beyond the standards. [For an analogy, I mention a long-standing phenomenon with some standardized tests—a seventh grader might be told that he or she has preformed at, say, the tenth-grade level. This does not mean that the seventh grader knows tenth-grade mathematics, but rather that he or she does as well on seventh-grade material as would an average tenth grader.]

Among the problems I include below is one which on first reading might seem appropriate for an MCA, but which nevertheless would not be. I point out, for instance, how it might have an unintentional cultural bias and thus would be bad for an MCA although possibly very good for classroom discussion.

The variety of different problems that are consistent with the standards and benchmarks is very large—that is the power of mathematics; a manageable number of basic principles and techniques enables one to handle a myriad of different situations. So, of course, the problem list that follows cannot be viewed as comprehensive.

For each problem in which students are to place the correct digits in boxes, I have included three boxes, or six boxes in case two sequences of digits are to be supplied. If the answer requires only two digits or one, it is the left-hand box or boxes which should be left blank. [If the Grade-4 MCA were, in fact, to include such problems it would be important that students become familiar with the instructions some days in advance of the test.]

It is clear from the benchmarks that the Grade-4 MCA should consist mostly of problems for which a calculator should not be permitted. The small latter portion of the problem list below is there in case the state decides that there should also be a 'calculator permitted' portion of the Grade-4 MCA; the place where this latter portion begins is clearly identified with an introductory sentence.

There is not a sharp demarcation separating problems appropriate for various grade levels. For instance, some of the problems described below as appropriate for Grade 4 are also in the link for Grade 3. Typically, a problem that is appropriate for both the Grade-4 MCA and the Grade-3 MCA would be regarded as a more difficult problem for a third grader than it would be regarded for fourth graders.

I want to again emphasize: Although the standards and the benchmarks accompanying them constitute an official document of the state of Minnesota, all the judgments about alignment of problems with the benchmarks and standards are mine; neither do they have any official standing nor have they been obtained in consultation with the Minnesota Department of Education. Also, they have not been reviewed by the University of Minnesota where I am a faculty member and, of course, they do not represent any official view of that institution.

3-II.B.2
4-I.2
4-I.5

Deborah had 133 pieces of candy, but he gave 84 pieces away. Which of the following equals the number of pieces of candy he still has?

(a) $133 - 84$
(b) $133 - 49$
(c) $133 + 49$
(d) $133 + 84$

Difficulty: standard-e. The benchmark 4-I.2 is involved even though there is no irrelevant information in the problem; there is irrelevant information in the detractors—the result 49 obtained by subtracting 84 from 133 is irrelevant.

3-V.C.3

Which of the following is correct?

(a) 12 feet equals 1 yard
(b) 12 yards equals 1 foot
(c) 12 feet equals 1 inch
(d) 12 inches equals 1 foot

Difficulty: standard-e.

4-II.A.2

Which of the following numbers is the largest?

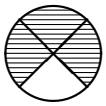
(a) $143,552,784$
(b) $54,999,333$
(c) $65,400,009$
(d) $973,256$

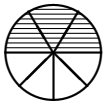
Difficulty: standard-e. A more difficult problem would request the second largest among these four numbers.

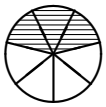
3-II.A.4
3-IV.A.1

Which of the following pictures shows the fraction 3/4?

(a) 

(b) 

(c) 

(d) 

Difficulty: standard-e. The correct answer is (b), but one could make a case for (d) if one were thinking of a setting involving proportions. However, at Grade 4 I think the problem is unambiguous. I did consider replacing the word 'shows' with the phrase 'best represents', but I decided against doing so since inclusion of the phrase 'best represents' might place the problem beyond the reading level of some fourth graders.

4-III.B.1
4-II.B.6
4-II.B.4

Fill in the blank: $56 \div \_\_\_\_ = 7$.

(a) 7
(b) 8
(c) 9
(d) 49

Difficulty: standard-h.

4-III.B.2
4-I.1

Which of the following is correct?

(a) Addition of 0 and multiplication by 0 always give the same result.
(b) Addition of 0 and multiplication by 1 always give the same result.
(c) Addition of 1 and multiplication by 0 always give the same result.
(d) Addition of 1 and multiplication by 1 always give the same result.

Difficulty: substantial. The statements in this problem are not as precise as they might be; making them more precise would also make them more clumsy—something to consider when writing MCA's.

4-II.A.4
4-I.4
4-I.2

Eileen has bought 19 toy cars during the last 6 months. Each one cost between $2.95 and $3.08. She has spent approximately

(a) $20
(b) $40
(c) $60
(d) $360

Difficulty: substantial. The number of months is extraneous information which the student has to decide to ignore. Also, the meaning of 'each' is a bit subtle at the Grade-4 level. Neither of the above aspects of the problem render it unusable on the Grade-4 MCA; they just contribute to its level of difficulty. However, I would be concerned that the amounts of money involved for toy-buying might seem reasonable for some children and unthinkable for other children. My view is that real-world problems that seem realistic to some children and very unrealistic to others can introduce significant unintended cultural bias into a test.

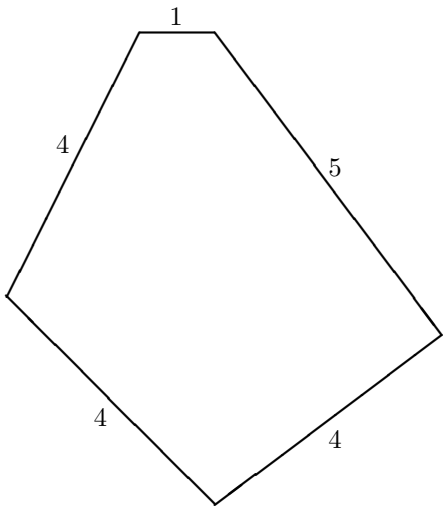
4-V.B.1
4-I.1

How many edges does a tetrahedron have? (Recall that a tetrahedron is a solid having four triangular faces.)

(a) 3
(b) 4
(c) 6
(d) 8

Difficulty: challenging. Would the problem be legal were the parenthetical remark about the meaning of 'tetrahedron' removed? The word 'Identify' in benchmark 4-V.B.1 indicates that the answer is 'yes'. On the other hand, I know from my stint on the 9-12 math subcommittee that the consensus there leaned away from an emphasis on the memory of names of figures in geometry. In any case I would favor including the parenthetical remark, for otherwise, the testing of the name gets confounded with the other aspect of the problem. Nevertheless, this problem will tend to be easier for a student who is familiar with the name 'tetrahedron' than for one who needs to read the parenthetical remark with care.

3-V.C.2 Find the perimeter of the polygon shown below.



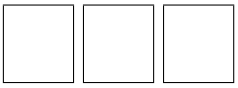
Difficulty: standard-e.

4-II.A.1 Write the number two-hundred seven as a numeral.

Difficulty: standard-e.

4-II.B.2 Calculate

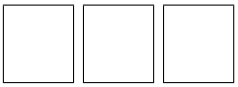
$$\begin{array}{r} 243 \\ +\ 532 \end{array}$$



Difficulty: standard-e.

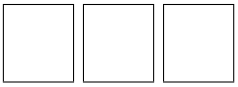
4-II.B.3 Calculate

$$\begin{array}{r} 369 \\ -\ 82 \end{array}$$



Difficulty: standard-h.

3-V.C.5 How many minutes is it from 11:48am to 12:27pm of the same day?



Difficulty: standard-h.

4-II.B.2 Calculate the following sum:

$$\begin{array}{r} 345 \\ +\ 479 \\ +\ 158 \\ \hline \square\square\square \end{array}$$

Difficulty: standard-h.

4-II.B.2
4-II.B.3
4-I.5
4-I.2

Alphonso attended school 166 days as a first grader and 165 days as a second grader. How many days does he have to attend as a third grader in order that his total attendance for the three grades equal 500 days?

$$\square\square\square$$

Difficulty: standard-h. This problem would also be appropriate Grade-3 MCA problem, provided that it were to appear on a calculator-permitted portion. At the Grade-4 level it is appropriate for a no-calculator portion.

4-I.5
4-III.B.2
4-II.B.4
4-II.B.1

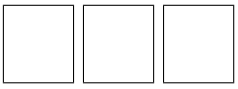
Add 6 to the product of 27 and 1. Then subtract that answer from the product of 9 and 5.

$$\square\square\square$$

Difficulty: substantial. One might be concerned that this problem is unnecessarily difficult for those for whom English is a second language. My judgment is that it is ok. It is not the vocabulary that makes this problem substantial. Rather the issue is the manner in which the simple words indicate an order of operations, and this somewhat subtle idea needs to be taught to all students since it doesn't seem to come naturally to most students whatever their first language happens to be.

4-I.5
2-V.C.4
4-II.B.1
4-II.B.2
4-II.B.3

Allison had saved three quarters, four dimes, two nickles, and six pennies, but then spent 57¢ for a toy. How many cents did she have left?



Difficulty: substantial. I have rated the difficulty high for Grade 4 because there are three aspects to the problem: reading it with care, converting coin values into cents, and a subtraction which involves two borrowings. In addition, the solution requires considerable care to avoid a mistake. I would not like this problem to be a multiple-choice problem since then the subtraction could be avoided by checking four additions, a checking skill which is useful but not the same skill as that of doing subtraction accurately. Notice that it is possible for Allison to pay exactly 57¢ with her own coins—thus, the issue of making change does not cloud the issue. When reading standard 3-II.B and benchmark 3-II.B.2 one might think that the phrase 'Compute fluently' in standard 3-II.B implies that this problem could be viewed as an ok, but hard, problem for the Grade-3 MCA. However, when one looks at benchmark 4-II.B.3, one infers that this problem would only be ok for the Grade-3 MCA if it were in a portion of the MCA for which calculators are permitted.

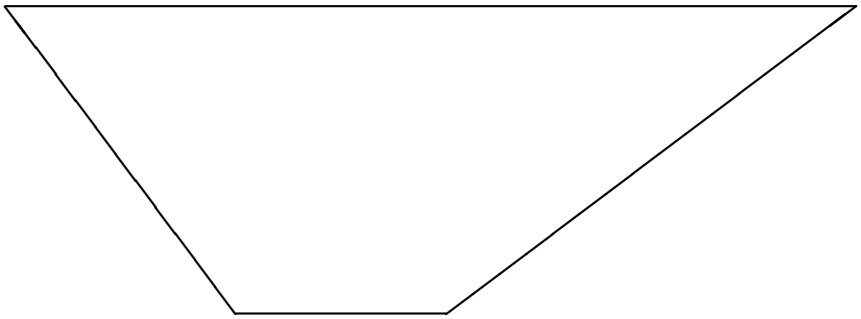
4-I.5
4-II.A.4
4-II.B.5
4-I.4

How many dozen eggs are required to feed 9 people, 2 eggs per day for a total of 5 days and how many eggs will be left over? The number of dozens should be the smallest number of whole dozens required and this number should be written in the left-hand set of boxes. The number of eggs left over should be written in the right-hand set of boxes.

Difficulty: challenging. Any problem requiring rounding or estimation at the level of Grades 3-5 is likely to be at least moderately difficult since the statement of the problem is likely to be a bit complicated in order that it not be ambiguous.

3-V.B.2
4-I.6

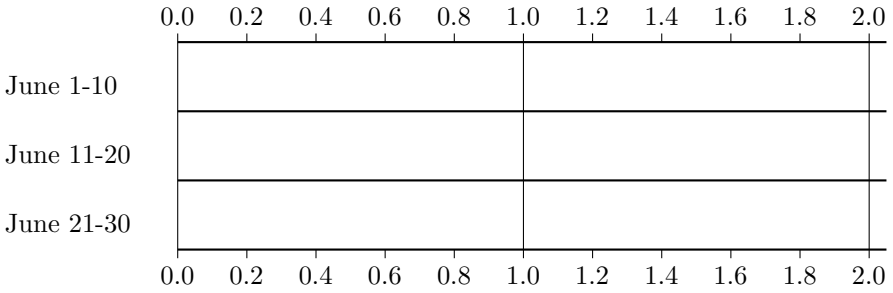
Show how the following shape is composed of two triangles and one rectangle. Label the parts of your picture.



Difficulty: standard-e. The problem does not refer to the figure as a trapezoid—that is fine for Grade 4. Were this a high school or possibly a middle school problem, something would have to be said to assure the student that the top and bottom edges are actually parallel rather than just close to parallel. But at such higher grade levels one can describe the problem more precisely without creating an unreasonable hurdle for students.

4-II.A.3
4-IV.A.1

During one June, the precipitation amounts in Phoenix, Arizona were 0.4 inches during the first ten days of June, 1.3 inches from June 11 through June 20, and 0.7 inches during the last ten days of the month. Illustrate this data with a bar graph.



Difficulty: Standard-h. I have not checked whether the data in this problem is realistic, which is an important consideration for an actual MCA problem. A state testing issue: There are various schemes for bar graphs and histograms. This problem, as laid out, forces the student to have horizontal bars with no space between the bars, a scheme that might be familiar to some but not to others. Were a state MCA to include a problem along these lines it would be wise to give the student a choice for layout, so the student could use the scheme with which he or she is most familiar. Another state testing issue: The level of difficulty of this problem depends on how familiar a student is with the word 'precipitation'.

4-V.C.2
4-I.6

Sketch pictures of two rectangles each having area 30, but having different perimeters. Indicate the side lengths in your pictures and calculate the perimeters of both rectangles.

Difficulty: substantial. This problem is somewhat difficult because the student is asked to do several things. An issue: ok to give an area without units? An alternative is to give the units as square centimeters. If one does this, one might then also ask for rather accurate pictures using a metric ruler.

FOR GRADE 4, WITH COMMENTS

If it can be managed, say with a voice-activated machine, I can envision part of the **MCA's for Grade 4 as being oral.** A problem might ask for a student to do as many of the simple calculations as he or she can in 2 minutes, with scoring based on the total number tried, total number correct, and total number incorrect. If such testing cannot be managed, then several problems similar to those below should appear on the Grade-4 MCA's, preferably as problems where the student has to fill in boxes with digits of the correct answers.

4-II.B.4
4-II.B.6
3-II.B.4
3-II.B.5

$7+9$	$6\div 2$	6×2	$13-9$
6×8	$8-0$	$7+3$	$12-6$
$0\div 4$	9×7	0×7	$5\div 5$
$12\div 3$	$9+6$	1×7	$7-7$
$7\div 1$	$2+8$	$5+7$	$11-4$
$18-9$	$24\div 4$	$3+3$	1×1
$7+2$	4×8	$6\div 3$	$2\div 2$
$18\div 6$	$5+5$	$4+6$	$7-6$
$2+0$	3×0	$0\div 9$	$64\div 8$
$3+8$	$9-2$	$56\div 8$	$56\div 7$
$9\div 3$	9×4	$7-4$	$35\div 7$
$8\div 4$	6×6	$6-6$	$2+5$
2×7	7×2	$11-9$	6×5
$32\div 4$	$16\div 2$	$15\div 3$	5×5
$45\div 5$	$6+4$	$6-4$	$5+5$

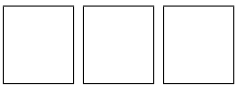
FOR GRADE 4, WITH COMMENTS

I include below some problems which might be considered were there to be a **calculator portion of the Grade-4 MCA**.

4-II.B.1 In one particular year, the precipitation amounts recorded for the city of Buffalo, New York were as follows:

January:	3 inches
February:	2 inches
March:	5 inches
April:	12 inches
May:	10 inches
June:	7 inches
July:	5 inches
August:	5 inches
September:	8 inches
October:	5 inches
November:	5 inches
December:	7 inches

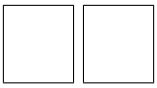
What was the precipitation amount in inches for the entire year?

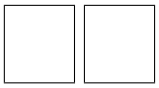


Difficulty: standard-h. I have not checked if the above amounts are realistic for Buffalo. But if this were a real MCA problem it would be important that they be realistic; for if they are not and some student happens to know they are not, then that student might suspect some kind of trick in the problem. One might be concerned that such a student will have an advantage in case the numbers are realistic—such an advantage will be slight for that knowledge will not give the exact answer.

4-I.5
2-V.C.3
4-II.B.1
4-II.B.5

For any year that is not a leap-year, find the one date that is in the center of that year—that is, find the date that has the same number of dates preceding it as following it. Enter the month in the left-hand array of boxes and the appropriate date within that month in the right-hand array of boxes. [For instance, if that date were March 29, you would enter either 3 or 03 in the left-hand boxes and 29 in the right-hand boxes.]





Difficulty: challenging. This problem is a good classroom problem, but it would not be appropriate for an MCA. Reason: If students have seen the problem before, they might remember the answer 07-02. It is great if students have learned a methodology in the classroom that happens to be useful on an MCA test aligned with the standards. That is quite different from happening to have memorized the interesting answer to an interesting question that might have been treated in some, but not all, classrooms. There is another issue with this problem as an MCA problem; some recent immigrants come from a culture in which the date is written in front of the month.