

PROBLEMS BASED ON THE SPRING 2003  
MATHEMATICS STANDARDS AND BENCHMARKS  
FOR GRADE 5  
WITH COMMENTS

by Bert Fristedt  
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The purpose of this link from my web-site is to identify a selection of problems aligned with the Minnesota mathematics standards and benchmarks for Grade 5 as adopted in Spring 2003. My focus consists of the standards and benchmarks themselves; the problems here serve to illuminate them. The benchmarks and standards that are particularly relevant for a particular problem are identified in the left-hand margins; for instance, 5-V.C.1 indicates the Grade-5 benchmark V.C.1 and 5-V.C refers to the corresponding standard. In another sense, the focus is the suitability of problems for the Minnesota Comprehensive Assessments (know as MCA's), but in saying this I want to emphasize that the opinions are mine alone, formed without consultation with Minnesota Department of Education. This particular link also includes a variety of comments about the problems. A problem list without this commentary is on another link.

I was one of approximately 40 members of the mathematics subcommittee of the Academic Standards Committee, formed by the Minnesota Commissioner of Education in February 2003. I strongly support the mathematics standards and benchmarks resulting from the work of that committee and which, on the basis of a law passed by the Legislature and signed by the Governor, became official in Spring 2003. Although there is no guarantee that this web-site item reflects the thinking within the Department of Education, I have tried very hard to reflect the standards and benchmarks accurately, taking care not to bend them in the direction of my individual views. [Even though I strongly support the standards and benchmarks document, there are places where I would have preferred the document to be a bit different, and I suspect that the same is true (but not for the same places) of every member of the mathematics subcommittee.]

Anticipating that I might want to modify this document from time to time, I have refrained from labeling the problems with numerals and am planning to change the date at the top any time I make additions or changes.

Since the standards are cumulative, all the K-5 benchmarks are relevant for the Grade-5 MCA. It seems to me that it is desirable for Grade-5 teachers to examine all the K-5 benchmarks giving special attention to those for grades 3-5, and in general for teachers to read the standards for a couple grades on either side of the grade they are teaching.

Even though I view all the problems below as consistent with the Grade-5 standards and benchmarks, the range of difficulty represented by them is wide. I have chosen the adjectives 'standard', 'substantial' and 'challenging' for the problems. The challenging problems are those that, in my opinion require sig-

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nificantly more than mastery of individual benchmarks. A substantial problem is one that has a feature of richness beyond what would be expected in a standard problem and yet which comes short of being the challenge that a challenging problem would represent. Among the standard problems I use two labels: standard-e, and standard-h indicating a distinction between easier and harder standard problems. This assignment of level of difficulty will follow each problem. But these personal opinions of mine are of secondary importance compared to the central issue of alignment of problems with benchmarks. I want to emphasize that the challenging and substantial problems are aligned with the benchmarks; it is not that they are on topics that go beyond the standards. [For an analogy, I mention a long-standing phenomenon with some standardized tests—a seventh grader might be told that he or she has preformed at, say, the tenth-grade level. This does not mean that the seventh grader knows tenth-grade mathematics, but rather that he or she does as well on seventh-grade material as would an average tenth grader.]

Among the problems I include below are some which on first reading might seem appropriate for an MCA, but which nevertheless would not be. I point out, for instance, how one problem might have an unintentional cultural bias and thus would be bad for an MCA although possibly very good for classroom discussion.

The variety of different problems that are consistent with the standards and benchmarks is very large—that is the power of mathematics; a manageable number of basic principles and techniques enables one to handle a myriad of different situations. So, of course, the problem list that follows cannot be viewed as comprehensive.

For problems in which students are to place the correct digits in boxes, a decimal point or comma is included between appropriate pairs of boxes when relevant. If the answer requires fewer digits than boxes, it is the left-hand box or boxes which should be left blank. [If the Grade-5 MCA were, in fact, to include such problems it would be important that students become familiar with the instructions some days in advance of the test.]

It is clear from the benchmarks that the Grade-5 MCA should consist mostly of problems for which a calculator should not be permitted. The small latter portion of the problem list below is there in case the state decides that there should also be a ‘calculator permitted’ portion of the Grade-5 MCA; the place where this latter portion begins is clearly identified with an introductory sentence.

There is not a sharp demarcation separating problems appropriate for various grade levels. For instance, some of the problems described below as appropriate for Grade 5 are also in the link for Grade 4. Typically, a problem that is appropriate for both the Grade-5 MCA and the Grade-4 MCA would be regarded as a more difficult problem for a fourth grader than it would be regarded for fifth graders.

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I want to again emphasize: Although the standards and the benchmarks accompanying them constitute an official document of the state of Minnesota, all the judgments about alignment of problems with the benchmarks and standards are mine; neither do they have any official standing nor have they been obtained in consultation with the Minnesota Department of Education. Also, they have not been reviewed by the University of Minnesota where I am a faculty member and, of course, they do not represent any official view of that institution.

- 4-II.A.2** Which of the following numbers is the second largest among the four numbers?
- (a) 143, 552, 784
  - (b) 54, 999, 333
  - (c) 65, 400, 009
  - (d) 973, 256

Difficulty: standard-e.

- 4-II.A.4**  
**5-I.4**  
**5-I.2** Eileen has bought 19 toy cars during the last 6 months. Each one cost between \$2.95 and \$3.08. She has spent approximately
- (a) \$20
  - (b) \$40
  - (c) \$60
  - (d) \$360

Difficulty: standard-e. The number of months is extraneous information which the student has to decide to ignore. I would be concerned that the amounts of money involved for toy-buying might seem reasonable for some children and unthinkable for other children. My view is that real-world problems that seem realistic to some children and very unrealistic to others can introduce significant unintended cultural bias into a test.

- 5-II.A.3** Which of the following fractions equals 75%?
- (a)  $\frac{3}{4}$
  - (b)  $\frac{4}{5}$
  - (c)  $\frac{7}{5}$
  - (d)  $\frac{7}{4}$

Difficulty: standard-e. A harder, but fair, version of this problem would not have had option (a) available, but would have had, say,  $15/20$  as one of the options (and the other options would possibly also be different).

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- 5-V.B.4** The sum of the measures of the angles in a triangle equals
- (a)  $60^\circ$
  - (b)  $90^\circ$
  - (c)  $120^\circ$
  - (d)  $180^\circ$

Difficulty: standard-e.

- 5-II.A.1** Which of the following numerals represents forty-five and twenty-three thousandths?
- (a) 0.4523
  - (b) 45.0023
  - (c) 45.023
  - (d) 45.23

Difficulty: standard-h. Notice that 0.068 is not listed as an option. Some might claim that a legitimate interpretation of the problem is that ‘forty-five’ and ‘twenty-three’ are both adjectives modifying ‘thousandths’, and that the conjunction ‘and’ denotes addition. Such a person should, however, be aware of the other possible interpretation that 45 is the whole number part of a decimal numeral, and then use this interpretation since 0.068 is not a given option. [An irregularity in names of numerals that did not arise in this problem, but of which all teachers should be aware, especially those who teach students for whom English is a second language: one billion in American English means  $10^9$ —that is, one thousand millions; whereas in British English it means  $10^{12}$ , which is one million millions. And of course, a similar issue arises in connection with one trillion.]

- 5-V.A.1** Which of the following is correct?
- (a) Two similar triangles are not necessarily congruent.
  - (b) Two congruent triangles are not necessarily similar.
  - (c) Two similar triangles are never congruent.
  - (d) Two congruent triangles are never similar.

Difficulty: standard-h. This problem should not be used on an MCA because it has a subtle flaw. Although there is only one correct response—namely, (a)—responses (c) and (d) each also imply another on logical grounds. If (c) were the correct response, then (a) would also be correct. Similarly for (d) and (b). After writing this problem and then deciding it was a bad problem, I decided to keep it in this list in order to point out this type of flaw.

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- 4-V.B.1**      How many edges does a tetrahedron have? (Recall that a tetrahedron is a solid  
**5-I.1**            having four triangular faces.)
- (a) 3
  - (b) 4
  - (c) 6
  - (d) 8

Difficulty: standard-h. Would the problem be legal were the parenthetical remark about the meaning of ‘tetrahedron’ removed? The word ‘Identify’ in benchmark 4-V.B.1 indicates that the answer is ‘yes’. On the other hand, I know from my stint on the 9-12 math subcommittee that the consensus there leaned away from an emphasis on the memory of names of figures in geometry. In any case I would favor including the parenthetical remark, for otherwise, the testing of the name gets confounded with the other aspect of the problem. Nevertheless, this problem will tend to be easier for a student who is familiar with the name ‘tetrahedron’ than for one who needs to read the parenthetical remark with care.

- 5-V.C.3**      Which is correct?
- (a) ‘Acres’ and ‘square feet’ are both appropriate units for area.
  - (b) ‘Acres’ is an appropriate unit for length and ‘square feet’ is appropriate for area.
  - (c) ‘Acres’ is an appropriate unit for area and ‘square feet’ is appropriate for volume.
  - (d) ‘Acres’ is an appropriate unit for time and ‘square feet’ is appropriate for area.

Difficulty: standard-h. The difficulty is mostly in the reading of the problem. There might be a concern that this problem favors rural students who have used the term ‘acres’ more in their lives than have students in cities. I think the problem is ok, since the term ‘acre’ is a mathematical term.

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4-III.B.2  
5-I.1

Which of the following is correct?

- (a) Addition of 0 and multiplication by 0 always give the same result.
- (b) Addition of 0 and multiplication by 1 always give the same result.
- (c) Addition of 1 and multiplication by 0 always give the same result.
- (d) Addition of 1 and multiplication by 1 always give the same result.

Difficulty: standard-h. The statements in this problem are not as precise as they might be; making them more precise would also make them more clumsy—something to consider when writing MCA's.

5-V.B.1

A pentagonal prism has pentagons on the top and the bottom and five rectangles on the sides. How many edges does it have?

- (a) 10
- (b) 15
- (c) 20
- (d) 30

Difficulty: standard-h. Even the simplest problem in three dimensions should get a difficulty rating of at least standard-h at the fifth-grade level. This problem has been stated in a manner that makes it accessible to a student who has forgotten (or never learned) the definition of 'prism', but it is a bit easier for the student who does know the definition because then the redundancy in the problem can give a boost in self-confidence.

5-V.A.1  
5-V.B.3  
4-V.B.1

How many reflective symmetries does a regular hexagon have? (Recall that a hexagon is a figure having six edges and six vertices.)

- (a) 0
- (b) 3
- (c) 6
- (d) 12

Difficulty: substantial.

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**4-II.B.2** Calculate the following sum:

$$\begin{array}{r} 345 \\ + 479 \\ + 158 \end{array}$$

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Difficulty: standard-e.

**5-II.B.3** Calculate

$$4.89 - 3.73$$

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Difficulty: standard-e. This problem is designed to be among the easier standard problems—no borrowing and both numbers have the same number of digits to the right of the decimal point and also the same number of digits to the left of the decimal point.

**5-II.B.4**  
**5-II.A.4** Calculate

$$\begin{array}{r} 37 \\ \times .79 \end{array}$$

and round your answer to the nearest whole number.

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Difficulty: standard-h.

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**5-II.B.1**  
**5-II.B**  
**5-II.B.3**

In one particular year, the precipitation amounts recorded for the city of Buffalo, New York were as follows:

January: 3 inches  
February: 2 inches  
March: 5 inches  
April: 12 inches  
May: 10 inches  
June: 7 inches  
July: 5 inches  
August: 5 inches  
September: 8 inches  
October: 5 inches  
November: 5 inches  
December: 7 inches

What was the precipitation amount in inches for the entire year?

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Difficulty: standard-h. I have not checked if the above amounts are realistic for Buffalo. But if this were a real MCA problem it would be important that they be realistic; for if they are not and some student happens to know they are not, then that student might suspect some kind of trick in the problem. One might be concerned that such a student will have an advantage in case the numbers are realistic—such an advantage will be slight for that knowledge will not give the exact answer. My judgment is that the standard 5-II.B in combination with the benchmark 5-II.B.3 implies that this problem is fine as a non-calculator problem for the Grade-5 MCA.

**5-I.2**  
**5-II.B.4**  
**5-II.B.1**  
**3-V.C.5**

How many minutes is it from 11:48am to 1:27am of the next day?

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Difficulty: standard-h.

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**5-I.5**  
**2-V.C.3**  
**4-II.B.1**  
**4-II.B.5**

For any year that is not a leap-year, find the one date that is in the center of that year—that is, find the date that has the same number of dates preceding it as following it. Enter the month in the left-hand array of boxes and the appropriate date within that month in the right-hand array of boxes. [For instance, if that date were March 29, you would enter either 3 or 03 in the left-hand boxes and 29 in the right-hand boxes.]

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Difficulty: substantial. This problem is a good classroom problem, but it would not be appropriate for an MCA. Reason: If students have seen the problem before, they might remember the answer 07-02. It is great if students have learned a methodology in the classroom that happens to be useful on an MCA test aligned with the standards. That is quite different from happening to have memorized the interesting answer to an interesting question that might have been treated in some, but not all, classrooms. There is another issue with this problem as an MCA problem; some recent immigrants come from a culture in which the date is written in front of the month. However, this issue is not critical; the grading could give full credit for either 07-02 or 02-07 (and also for modifications such as 7-2 or 02-7).

**5-I.5**  
**5-II.B.1**  
**5-II.B.5**  
**5-II.B.3**

A pound of cotton has been spun into a thread 8 miles in length. Allowing for 235 pounds of waste, how many pounds will it take to spin a thread to reach around the earth, supposing that distance to be 25,000 miles?

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Difficulty: substantial. [This problem comes from a grades-5-8 book first published in 1858 and reprinted in 1863 and 1877 (which is the date of my copy). This book was authored by Daniel W. Fish and is entitled ‘Robinson’s Progressive Practical Arithmetic.’] Although 25,000 has more than 3 digits, it still fits within the scope of benchmark 5-II.B.5 because its last three digits are zeroes. Note: It is obvious, from reading all the words after the first word in benchmark 5-II.B.5, that the first word of that benchmark should be ‘Divide’ not ‘Multiple’. Some might say that this problem is not appropriate for the Grade-5 MCA, since it is a ratio-proportion type problem. But the fact the number of pounds of cotton in the given information is one means that the problem is not a full ratio-proportion type problem. Change the number one to three and the problem becomes inappropriate for the Grade-5 MCA.

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5-IV.A.2  
 5-II.B.6  
 5-II.A.2  
 5-II.A.3

What percentage of the following numbers are greater than  $-3\frac{2}{5}$ :

$-7$ ,  $7$ ,  $-7/2$ ,  $7/2$ ,  $-3.72$ ,  $3.72$ ,  $-3$ ,  $3$ ,  $-2/7$ ,  $2/7$ ?

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Difficulty: substantial. There is a bit of a glitch in the standards and benchmarks. Mixed numbers are mentioned in benchmark 5-II.B.6, but nowhere else under standards 5-II.A and 5-II.B. Of course, fractions can represent mixed numbers, but it would have been better had mixed numbers been mentioned explicitly. Nevertheless, I think this problem involving a mixed number is certainly in the spirit of the Grade-5 standards and benchmarks.

5-I.2  
 5-II.B.1  
 5-II.B.5  
 4-II.B.7  
 5-II.B.3

A man purchased a house for \$118,750, and expended \$17,000 in repairs; he then sold it for railroad stock worth \$43,350, and 235 acres of western land valued at \$400 an acre. What dollar amount did he gain by the trade?

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Difficulty: substantial. The multiplication of 235 by 400 involves handling of the two zeroes in combination with multiplication of a three-digit number by a one-digit number. My judgment is that this combination of skills is encompassed by benchmarks 5-II.B.5 and 4-II.B.7. Also relevant is the the standard 5-II.B itself. Benchmark 5-II.B.4 has a limit of five digits, but I consider this problem ok in this regard since the last digit is 0 in any number that is to added to or subtracted from \$118,750. This being said, I would regard this problem as inappropriate for an MCA. The colorful adjective ‘western’ for the noun ‘land’ can spark the imagination of some, while being nothing but a puzzle for others. But it is fine for in-class discussion. [This problem comes from the ante-bellum book by Fish mentioned earlier, except that the dollar amounts here have been obtained by multiplying those in the book by 50.]

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**5-II.B.6**

If  $5\frac{1}{9}$  yards of cloth are required for a coat,  $3\frac{1}{6}$  yards for a matching pair of pants, and  $\frac{7}{9}$  yards for a vest, how many yards will be required for all three items of clothing? Write the answer as a mixed number with the fractional part in lowest terms. Enter the numerator of the fractional part in the left-hand boxes and its denominator in the right-hand boxes.

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Difficulty: substantial. Actually, this problem is inappropriate for any MCA for two reasons: (i) The non-mathematical word ‘vest’, which plays no essential role in the problem, is a word which might not be familiar to certain students. (ii) One would think that an appropriate unit for cloth is one that measures area. ‘Yards’ measures length, not area. The problem becomes clear only when one knows that length in one direction is what is important since the cloth comes off of a reel of standard width. This second reason for the problem not being appropriate for an MCA happens to be a good reason for using the problem in a class discussion. Also, there is the issue of whether  $1/9$  and  $7/9$  would be called common fractions as specified in benchmark 5-II.B.6. My judgment is that they should be. Clearly benchmark 5-II.B.6 does encompass the concept of common denominator. The restrictions in the benchmarks are there to limit the computation needed to find a common denominator. In this problem, that computation is indeed limited. This is a problem for which partial credit could be given for a correct answer not in lowest terms. [This problem comes from the ante-bellum book by Fish mentioned earlier.]

**5-IV.A.4**  
**5-I.5**  
**5-II.B.5**

During one June, the precipitation amounts in Phoenix, Arizona were 0.4 inches during the first ten days of June, 1.3 inches from June 11 through June 20, and 0.7 inches during the last ten days of the month. Calculate the mean daily rainfall correct to 3 places to the right of the decimal point.

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Difficulty: substantial. This problem requires use of the definition of *mean* in a setting where individual data are not given. The required division in the problem is by 30, but I regard that as falling under benchmark 5-II.B.5, since the second digit of 30 is zero.

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**5-II.B.5**

Calculate  $58.1 \div 0.07$ . *Hint:* The answer is a whole number.

□	□	□	□	□
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Difficulty: substantial. At the fifth-grade level, handling the decimal point in long division is a substantial task. The reason for giving the hint was to avoid either having to give instructions about how to enter a decimal point if needed or saying nothing but having a student worry about this in advance of doing the problem.

**5-II.B.5**

**5-I.5**

**5-I.2**

**5-II.B.1**

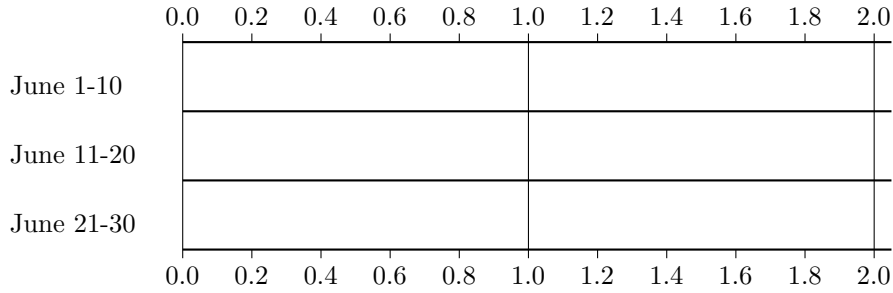
Points A and B are 12 miles apart. John bicycles from A to B at an average speed of 6 miles per hour and returns from B to A at an average speed of 4 miles per hour. What is John's average speed in miles per hour for the round trip: A to B and back to A? Write your answer as either a whole number or as a decimal, whichever is appropriate.

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Difficulty: challenging.

**4-II.A.3**  
**4-IV.A.1**

During one June, the precipitation amounts in Phoenix, Arizona were 0.4 inches during the first ten days of June, 1.3 inches from June 11 through June 20, and 0.7 inches during the last ten days of the month. Illustrate this data with a bar graph.



Difficulty: standard-e. I have not checked whether the data in this problem is realistic, which is an important consideration for an actual MCA problem. A state testing issue: There are various schemes for bar graphs and histograms. This problem, as laid out, forces the student to have horizontal bars with no space between the bars, a scheme that might be familiar to some but not to others. Were a state MCA to include a problem along these lines it might be wise to give the student a choice for layout, so the student could use the scheme with which he or she is most familiar. The actual given data in this problem is the same as that in an earlier problem. That would be fine for an in-class treatment. But on an MCA one wants to avoid situations where difficulty in an earlier problem can carry over, even psychologically, to a later problem.

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- 4-V.C.2**  
**5-I.6** Sketch pictures of two rectangles each having area 30, but having different perimeters. Indicate the side lengths in your pictures and calculate the perimeters of both rectangles.

Difficulty: standard-h. This problem is somewhat difficult because the student is asked to do several things. An issue: ok to give an area without units? An alternative is to give the units as square centimeters. If one does this, one might then also ask for rather accurate pictures using a metric ruler.

- 5-IV.B.1** A deck consisting of four cards labeled A, B, C, and D is shuffled and then the top two cards are drawn in order. Make a list describing the possible outcomes of this experiment.

Difficulty: standard-h. The difficulty would rise were the problem to request a clear explanation in full sentences of any symbolism that is used.

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**5-III.A.1**  
**5-I.7**

Consider the sequence of numbers

$2, 7, 22, 67, \dots$

Complete the following sentence: In the above sequence the initial term equals 2 and each subsequent term is obtained from the term preceding it by  $\dots$ .

Then find the term that comes immediately after 67.

Difficulty: challenging.

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I include below some problems which would be appropriate were there to be a **calculator portion of the Grade-5 MCA**.

**5-II.A.4**  
**5-II.B**

Find

$$(27.314 + 15.337) \times 2.1223.$$

The best 5-digit approximation of the answer equals

- (a) 59.863
- (b) 59.864
- (c) 90.518
- (d) 90.519

Difficulty: standard-h. One could consider giving partial credit for response (d).

**5-I.2**  
**5-I.7**  
**5-V.B.1**  
**4-V.A.1**  
**5-II.B.1**

A non-regular octahedron is positioned with one of its eight triangular faces on a floor. That triangular face and its opposite face are congruent equilateral triangles each having perimeter 17.19. The other edges of the octahedron are 1.87 times the length of the edges of the bottom face. Find the total length of all the edges of the octahedron.

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Difficulty: challenging. I am reluctant to make problems designed to reward the high achievers into multiple-choice questions, for then the randomness connected with guesses on such questions can actually play a non-informative role in the scores of middle- and low-achieving students. I have kept all four places to the right of the decimal point in order that estimation and rounding not be another aspect of this problem which already has very many aspects.

The views and opinions expressed in this link are strictly those of Bert Fristedt. The contents have been neither reviewed nor approved by the University of Minnesota.