

SOLUTIONS of problems
based on the mathematics standards document
FOR GRADE 7

by Bert Fristedt

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The purpose of this link from my web-site is to give answers to the problems in another of my links—the one entitled ‘Problems Based on the Spring 2003 Mathematics Standards and Benchmarks for Grade 7, with Comments’. The problems with comments are repeated here, but the rather lengthy preliminary discussion has been removed, and replaced by this and the next three paragraphs.

It is important to realize that I do not speak for the Minnesota Department of Education (MDE). Thus, the fact that I have posted these problems on my web site does not imply that MDE regards these problems as accurately reflecting the standards or as indicating the scope of the state tests (known as the MCA’s II).

While I do try to check my calculations carefully, these answers do not have the benefit of being checked by another person. Thus I cannot guarantee that they are 100% accurate.

The answers to the multiple-choice questions are indicated by the letter (Y) in place of (a), (b), (c), or (d). The answers to the fill-in problems are besides the boxes, not in them. And when there are separate sets of boxes for parts of the answers, the parts are not usually separated in the answers that I give. The MCA’s do not have fill-in-the-boxes formats for answers, but rather gridded response sheets.

FOR GRADE 7, with comments and ANSWERS

7-II.A.1 Which of the following mixed numbers equals the improper fraction $13/5$?

- (Y) $2\frac{3}{5}$
- (b) $3\frac{2}{5}$
- (c) $3\frac{2}{3}$
- (d) $10\frac{3}{5}$

Difficulty: standard-e.

7-II.B.1 $\frac{2}{3} \div \frac{4}{9} =$

- (a) $8/27$
- (b) $2/3$
- (Y) $3/2$
- (d) $27/8$

Difficulty: standard-e. This problem illustrates an advantage of some multiple-choice problems. The focus here is on being careful about what belongs in numerators and what belongs in denominators; it is clear that there will be no partial credit if one gets a fraction turned upside-down, while it gives a student a chance to correct an arithmetic error such as $9 \times 3 = 24$. The vertical and slanting methods of writing fractions are both used extensively, so it is important that students be familiar with both.

FOR GRADE 7, with comments and ANSWERS

- 7-II.A.1** $62.5\% =$
 (Y) $5/8$
 (b) $5/6$
 (c) $25/4$
 (d) $125/2$

Difficulty: standard-e

- 7-II.B.5** $5^3 =$
 (a) 75
 (Y) 125
 (c) 225
 (d) 243

Difficulty: standard-e.

- 7-V.C.1** Which of the following is the most appropriate physical unit with which to describe the capacity of the gasoline tank in an automobile?
 (a) centimeter
 (b) kiloliter
 (Y) liter
 (d) meter

Difficulty: standard-h. It is difficult to write a problem of this type that does not have cultural bias.

FOR GRADE 7, with comments and ANSWERS

- 6-III.A.1** In the rectangular coordinate system, as usually drawn, which of the following is true about the line through the points $(1, -2)$ and $(-1, -2)$:
- (Y) The line is horizontal.
 - (b) The line is vertical.
 - (c) The line is slanted with the lower part to the left and the higher part to the right.
 - (d) The line is slanted with the higher part to the left and the lower part to the right.

Difficulty: standard-h.

- 7-II.B.2**
4-V.C.1
7-V.B.4 Find the side-length of a square whose area is 729 square inches.
- (a) 9 inches
 - (Y) 27 inches
 - (c) 81 inches
 - (d) 521, 441 inches

Difficulty: standard-h. I usually do not like mathematics multiple-choice problems that are best solved by working with the multiple options. For this problem I make an exception because it is so natural to look for things to square when square roots are involved in solving a problem. The benchmark 4-V.C.1 does not explicitly speak of squares, only of rectangles. For this reason I have also listed 7-V.B.4 as a relevant benchmark. The central benchmark for this problem is, however, 7-II.B.2. I would regard this as a standard-e problem were it on the calculator portion of a Grade-7 state test.

- 7-III.A.2** Find the slope of the line in a plane that passes through the points $(4, 3)$ and $(-1, 1)$ with, as usual, the axis for first coordinates being the horizontal axis.
- (Y) $2/5$
 - (b) $3/4$
 - (c) $4/3$
 - (d) $5/2$

Difficulty: standard-h.

FOR GRADE 7, with comments and ANSWERS

7-V.B.3 On a drawing of the floor plan of a particular house, one foot in the house is represented by $\frac{1}{8}$ of an inch. How long is the actual house if the length as measured on the drawing is $4\frac{3}{4}$ inches?

- (a) 17 feet 6 inches
- (b) 19 feet
- (c) 35 feet
- (Y) 38 feet

Difficulty: standard-h.

7-II.B.4 What does 5% of 2% equal?

- (Y) 0.1%
- (b) 0.4%
- (c) 10%
- (d) 40%

Difficulty: substantial. This problem requires conversion to decimals and then back (unless one realizes that if one converts only one of the given numbers to decimals, then no further conversions are needed).

7-II.A.3 Which of the following statements is true?

7-II.B.1

- (a) $\frac{17}{12} < \frac{41}{29} < \frac{99}{70}$
- (b) $\frac{17}{12} < \frac{99}{70} < \frac{41}{29}$
- (c) $\frac{41}{29} < \frac{17}{12} < \frac{99}{70}$
- (Y) $\frac{41}{29} < \frac{99}{70} < \frac{17}{12}$

Difficulty: substantial. This problem has two aspects that contribute to its substantial difficulty: (1) the need to understand that one can compare rational numbers by comparing the products obtained by cross-multiplication, an understanding that might be vague even for someone who often essentially does the cross-multiplication when doing subtraction of fractions; (2) the lack of a clear starting place for the solution, even though, in fact, one can profitably start by comparing any two of the three given fractions.

FOR GRADE 7, with comments and ANSWERS

7-V.B.1
7-I.3
5-I.2

Find the area of the region between a circle of radius 5 and a circle of radius 2 lying inside the circle of radius 5.

- (a) 3π
- (b) 7π
- (Y) 21π
- (d) 29π

Difficulty: substantial.

7-IV.A.2
6-IV.A.2
7-I.2

Three positive numbers have the property that one of them is larger than the sum of the other two. Which of the following four assertions is accurate.

- (a) The mean of the three numbers will definitely be less than the median of the three numbers.
- (b) The mean of the three numbers will definitely equal the median of the three numbers.
- (c) The mean of the three numbers will definitely be larger than the median of the three numbers.
- (Y) Neither (a) nor (b) nor (c) is an accurate statement.

Difficulty: challenging. A correct approach is to consider two very different examples—one, for instance, in which the three numbers are 1, 4, and 6 and another in which they are 1, 4, and 100. Then simple calculations show that (d) is the correct response. However, at the Grade-7 level, finding this approach is a challenging task.

5-II.B.3

$7.96 + 4.6 =$

. 12.56

Difficulty: standard-e. This problem would be appropriate as a standard-h problem for a Grade-5 state test, but it is still a fine problem for a Grade-7 state test.

FOR GRADE 7, with comments and ANSWERS

6-II.B.5
6-II.B.4

Find the remainder for the division problem $735 \div 22$.

		9
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Difficulty: standard-e. There is a bit of difficulty in this problem because the problem solver has to realize what is being asked—the remainder in this case, not the quotient. It is important that the state distribute sample tests in order to make students familiar with the fact that statements of problems might request something less than that which would usually be called a full answer, which in this case would either be an integer quotient together with a remainder or a complete quotient as a mixed number.

7-II.B.1

Calculate $\frac{3}{4} \times \frac{5}{12}$, writing your answer in lowest terms. Enter the numerator in the left-hand boxes and the denominator in the right-hand boxes.

					5/16
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Difficulty: standard-e. Partial credit might be warranted for a correct answer not in lowest terms.

5-II.A.3
7-II.B.1

Take $\frac{11}{128}$ from $\frac{75}{128}$ and reduce the answer to lowest terms. Write the numerator in the left-hand boxes and the denominator in the right-hand boxes.

						1/2
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Difficulty: standard-e. [This problem comes from a grades-5-8 book first published in 1858 and reprinted in 1863 and 1877 (which is the date of my copy). This book was authored by Daniel W. Fish and is entitled ‘Robinson’s Progressive Practical Arithmetic.’]

FOR GRADE 7, with comments and ANSWERS

7-II.B.3
7-II.B.4

Find the new price of an article if the old price of \$60 has been increased by 7%. Enter the whole dollar amount in the left-hand boxes and the cents in the right hand-boxes.

					64.20
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Difficulty: standard-e. No benchmark speaks directly to the issue in this problem, but the skill mentioned in 7-II.B.3 would usually be regarded as more difficult than the one needed here. Moreover, benchmark 7-II.B.4 indicates that a broad range of skills with numbers in various forms is to be expected.

6-II.B.4
4-III.B.2

Calculate 0.74×23.9 .

					17.686
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Difficulty: standard-h. Benchmark 4-III.B.2 is mentioned because the problem might be a bit easier if one views the given expression as 23.9×0.74 .

7-II.B.4
7-I.3
7-I.1
6-II.B.3

Alphonso is buying a desk and chair. Both are on sale. The original price of the desk was \$100, but there is 20% off due to the sale. The original price of the chair was \$60 with 15% off due to the sale. How much does Alphonso have to pay for the desk-chair combination? (Assume that there is no sales tax.)

\$						131.00
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Difficulty: standard-h. Benchmark 7-I.1 is mentioned because a realization that the answer must be significantly more than 100 dollars will help catch the error of confusing amount 'saved' with the amount paid.

FOR GRADE 7, with comments and ANSWERS

7-IV.B1
6-II.A.2
7-II.A.1

To the nearest whole percent, what is the probability, in percentages, of rolling a three with a perfectly balanced die?

17

Difficulty: standard-h.

7-I.3
6-II.B.3
7-I.1
5-II.B.5

A pound of cotton has been spun into a thread 8 miles in length. Allowing for 235 pounds of waste, how many pounds will it take to spin a thread to reach around the earth, supposing that distance to be 25,000 miles?

, 3,360

Difficulty: standard-h. [This problem comes from the ante-bellum book by Fish mentioned earlier.]

FOR GRADE 7, with comments and ANSWERS

7-II.B.1
7-II.B.8
6-II.B.2

Calculate

$$\frac{4}{21} - \frac{2}{35},$$

writing your answer in lowest terms. Enter the numerator in the left-hand boxes and the denominator in the right-hand boxes.

2/15

Difficulty: standard-h. Partial credit for 14/105 seems appropriate to me. The mental calculation benchmark has been included because the more comfortable one feels with doing arithmetic with the primes 3, 5, and 7 the more likely it is that one can do this problem quickly with small chance of error.

7-II.A.2
6-II.B.6

The quotient $(4 \times 10^4) \div (5 \times 10^2)$ can be written as a one-digit integer times a power of 10. Write the one-digit integer in the left-hand box and the power of 10 in the right-hand boxes.

8

1

Difficulty: standard-h.

7-III.B.3
7-III.B.1
7-II.B.5

Suppose x , y , and z are related by the formula $z = xy^2$. Find x when $y = 3$ and $z = 126$.

14

Difficulty: standard-h.

FOR GRADE 7, with comments and ANSWERS

6-II.B.8
6-II.A.1

What percentage of the following numbers are greater than $-3\frac{2}{5}$:

-7 , 7 , $-7/2$, $7/2$, -3.72 , 3.72 , -3 , 3 , $-2/7$, $2/7$?

70

Difficulty: substantial. The fact that different forms of numbers are used in the problem creates a substantial hurdle.

5-IV.A.4
7-I.3
7-I.5
6-II.B.4

During one June, the precipitation amounts in Phoenix, Arizona were 0.4 inches total for the period June 1 through June 10, 1.3 inches total from June 11 through June 20, and 0.7 inches total during the last ten days of the month. Calculate the mean daily rainfall correct to 3 places to the right of the decimal point.

. 0.080

Difficulty: substantial. This problem requires use of the definition of *mean* in a setting where individual data are not given. The student has to recognize that nevertheless there is sufficient information to do the problem.

6-II.B.2
6-II.B.1

What is the greatest common divisor of 468 and 1248?

156

Difficulty: substantial. [This problem comes from the ante-bellum book by Fish mentioned earlier.] The presence of this problem as a Grade-7 problem, even though it is specifically designed to fit one Grade-6 benchmark only, highlights the cumulative nature of the standards. For instance, if a Grade-7 teacher, when focusing on the standards, makes use of least common multiples (that is, least common denominators) for adding and subtracting fractions, then, by checking the Grade-6 benchmarks, he or she will realize that such a time is also ideal for reviewing greatest common divisors. I have had a hard time in deciding what level of difficulty to assign to this problem. On the one hand it is fully encompassed in a single Grade-6 benchmark—namely 6-II.B.2. But the numbers here are sufficiently large that the best approach is to first treat the problem as two non-trivial ‘find the prime factorization’ problems (see 6-II.B.1).

FOR GRADE 7, with comments and ANSWERS

7-II.B.4
7-II.B.1
7-II.B.8

Calculate

$$5\frac{5}{6} \times 2\frac{4}{5}$$

Write the answer as a mixed number with the fractional part in lowest terms. Then place the numerator of the fractional part of the answer in the left-hand boxes and the denominator of the fractional part in the right-hand boxes.

							16 $\frac{1}{3}$ (fractional part is 1/3)
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Difficulty: substantial. I would think partial credit would be warranted if all but the reduction to lowest terms is correct. Skill at mental arithmetic will enable a student to immediately change the problem to that of multiplying two improper fractions.

7-II.B.1
7-V.C

The perimeter of a certain square is $27\frac{13}{16}$ inches. Find the length of each side in inches, writing your answer as an improper fraction in lowest terms. Enter the numerator in the left-hand boxes and the denominator in the right-hand boxes.

							445/64
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Difficulty: substantial.

7-III.B.3
7-I.3
4-V.B.1
5-I.2

The formula for the surface area of a rectangular parallelepiped in which two faces are squares is $2(x^2 + 2xy)$, where x is the edge length of the square faces and y is the length of the other edges of the parallelepiped. Find y when the surface area equals 410 and $x = 5$. *Reminder:* A rectangular parallelepiped has six faces all of which are rectangles; and in this problem two of those rectangles happen to be squares. *Hint:* The answer is a whole number.

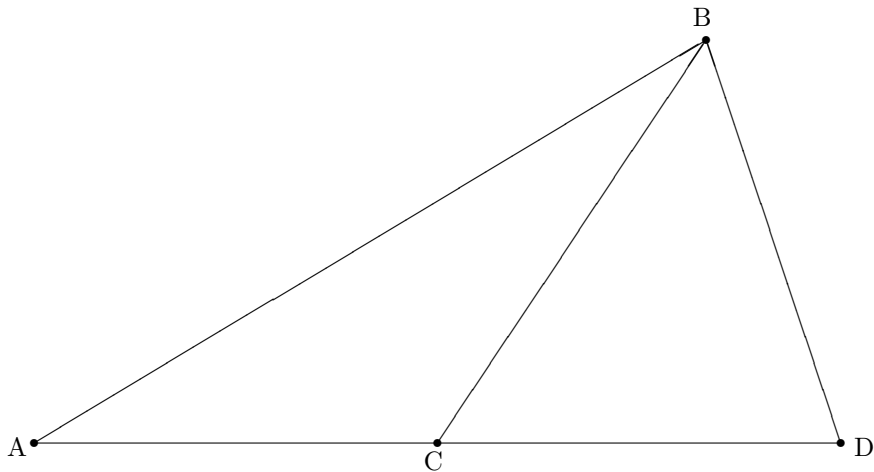
			18
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Difficulty: challenging. In this problem, no symbol for the surface area is given; if one had been given and used with an equals sign in the formula, the problem would be easier. To solve this problem one only needs to work with the given formula and the numbers 410 and 5. Of course to realize this, one has to sort through the terminology in the problem, but the relevant words are themselves related to the mathematics standards.

FOR GRADE 7, with comments and ANSWERS

6-V.B.1
6-V.B.2

In the following picture, the line through the points A and D passes through the point C , and both triangles are isosceles triangles. In the left-hand triangle the vertices at A and B each have measure 25° . In the right-hand triangle the vertices at B and C have the same measure. Calculate the measure in degrees of the vertex D . The picture has *not* been drawn accurately.



80

Difficulty: challenging.

7-II.A.1

Write $3.4\overline{87}$ as a fraction in lowest terms. [Note: the bar above 87 indicates that the decimal numeral goes on forever with the repeating pattern 87.] Enter the numerator in the left-hand boxes and the denominator in the right-hand boxes.

 1151/330

Difficulty: challenging. Even though only one benchmark is involved, the fact that the repeating part does not begin at the decimal point creates a significant complication at the Grade-7 level.

FOR GRADE 7, with comments and ANSWERS

6-IV.B.2

5-IV.B.1

7-I.6

A deck consisting of four cards labeled A, B, C, and D is shuffled and then the top two cards are drawn in order. Make a list describing the possible outcomes of this experiment. Then calculate the probability that A is the first card or D is the second card.

(A, B) , (A, C) , (A, D) , (B, A) , (B, C) , (B, D)
 (C, A) , (C, B) , (C, D) , (D, A) , (D, B) , (D, C)

Among the above 12 possible outcome, the first, second, third, sixth, and ninth outcomes have either A first or D second (or both). So, the answer is $5/12$.

Difficulty: substantial.

7-IV.B.2

A die is rolled three times in succession, and each of the three times a five is obtained. Some people might say that the probability of obtaining a five on the next role equals $1/6$, whereas others might say that this probability is significantly larger than $1/6$. Depending on one's point of view, one can support either of these assertions. Explain.

One could view the die as well-balanced and say the probability of five on the next role is $1/6$. Alternatively, one could take the view that we do not know how the die was constructed or from where it has come, so the succession of 3 fives is evidence that the die is not well-balanced and that the probability of a 5 is significantly larger than $1/6$.

Difficulty: challenging

FOR GRADE 7, with comments and ANSWERS

6-V.C.1
7-V.C
7-I.4
7-I.3
7-II.B.1

Here are two facts about liquid measure:

(i) one gill equals one-fourth of a pint;

(ii) one U.S. barrel equals $31\frac{1}{2}$ gallons.

How many gills are there in $\frac{2}{3}$ of a barrel?

4 gills to a pint, 2 pints to a quart, 4 quarts to a gallon, $\frac{63}{2}$ gallons to a barrel, but only $\frac{2}{3}$ of a barrel.

$$4 \times 2 \times 4 \times \frac{63}{2} \times \frac{2}{3} = 672 \text{ gills}$$

Difficulty: challenging. At the Grade-7 level it is a challenge to immediately make use of two facts with which one was previously unfamiliar in connection with previously memorized facts relating pints to quarts and quarts to gallons, and in addition to use $\frac{2}{3}$ correctly in the problem. [It is important for the Department of Education to inform schools about which facts should be memorized—I think the facts that one quart equals two pints and that one gallon equals four quarts should be on that list. Of course, the Department of Education could take the point of view that all such needed facts will be provided, but I think that would be contrary to the standards the tone of which gives the impression that some minimal memorization is to be expected.] Another aspect of the difficulty for the student is that a student might see dimly that there are several correct ways to start the problem, and thus will be concerned about picking the most appropriate; actually there are several correct approaches all involving about the same amount of work. Whichever of these correct methods a student uses, the student is expected to say enough so that the solution reader can follow the reasoning; thus benchmark 7-I.4 plays a significant role.

FOR GRADE 7, with comments and ANSWERS

The following problems are designed with the **calculator portion of the Grade-7 MCA** in mind.

7-II.B.6

Find

$$(27.314 + 15.337) \times 2.1223.$$

The best 5-digit approximation of the answer equals

- (a) 59.863
- (b) 59.864
- (Y) 90.518
- (d) 90.519

Difficulty: standard-e. One could consider giving partial credit for response (d).

7-II.B.6

With accuracy in two places to the right of the decimal point,

7-I.1

$$7.77 \times [3329.23 - (45872.1 - 955.25)]$$

equals

- (a) $-337,980.39$
- (Y) $-323,135.81$
- (c) $-20,959.23$
- (d) $-19,048.71$

Difficulty: standard-e.

7-II.B.5

7-II.B.6

7-II.B.7

7-I.1

7-II.B.8

Calculate 3×5^6 . *Reminder:* The four options below are sufficiently different that one can actually avoid using a calculator by instead making an estimate.

- (a) 23,328
- (Y) 46,875
- (c) 1,889,568
- (d) 11,390,625

Difficulty: standard-h. By making this a multiple-choice problem, one focuses on order of operations. Of course, it is very important that the three wrong choices all come from punching the correct calculator buttons in some incorrect order. The first three benchmarks are relevant for doing the problem by direct computation, and the last two are relevant if estimation is used.

FOR GRADE 7, with comments and ANSWERS

6-V.C.1
6-II.A.2

One mile equals 5280 feet. To the nearest one-hundredth of a mile, how many miles are 205,321 feet?

. 38.39

Difficulty: standard-e. Some partial credit might be warranted for 38.88 even though the correct answer 38.89. Notice that not enough space has been given for the answer in case the student multiplies by 5280 rather than divides by it; this fact makes the problem a bit easier than it might otherwise be.

6-V.B.3
6-V.C.1

The circumference of a certain circle equals 7 feet, 5 inches. Calculate its radius to the nearest inch. You may use the approximation 3.14 for π or you may use the key for π itself on your calculator.

14

Difficulty: standard-h.

7-II.B.2
6-V.C.3
7-I.4
7-I.1

To three decimal-place accuracy find the edge length in inches of a square whose area equals that of a rectangle whose edge length measurements are 5 inches and 12 inches.

. 7.746

Difficulty: standard-h. The benchmark 7-I.1 is appropriate since a student should anticipate getting an answer somewhere between, say, 7 and 10; so if he or she doesn't, then this student will know that the work should be checked.

FOR GRADE 7, with comments and ANSWERS

7-II.B.3

7-I.3

7-I.1

To what amount does \$3050 grow after 4 years 8 months at an annual simple interest rate of $5\frac{1}{4}$ per cent. Round your answer to nearest cent.

\$. 3797.25

Difficulty: standard-h. [This problem comes from the ante-bellum book by Fish mentioned earlier. Of course, the students then would have to do all the calculations by hand.] Actually, no rounding is necessary if one works with fractions; the answer is exact. If one approximates $4\frac{2}{3}$ by a decimal, then an error of several cents can be made depending on which approximation is used; partial credit for any answer within one dollar of the correct answer seems appropriate. Implicit in the problem is an assumption that the monthly interest rate is $\frac{1}{12}$ that of the annual interest rate, regardless of the number of days in the month; I think it is reasonable to expect a student to see that such an assumption is being made and thus that no attention need be given to which eight months of the fifth year are relevant.

6-II.B

Calculate the following sum to the nearest one-hundredth:

$$4.592 +^{-} 3.4449 + 4\frac{7}{17} +^{-} 2\frac{8}{9}$$

. 2.67

Difficulty: substantial. The presence of fractions in some numbers and decimals in others creates a complication. At the Grade-7 level, negative numbers also provide an additional hurdle. There is a notational issue connected with negative numbers, especially at Grades 6 and 7: Should the test distinguish between the minus symbol and the negative symbol by the height at which ‘−’ is printed?

FOR GRADE 7, with comments and ANSWERS

7-I.3
6-II.B.4
7-I.1

Points A and B are 17.2 miles apart. John bicycles from A to B at an average speed of 6.7 miles per hour and returns from B to A at an average speed of 4.3 miles per hour. What is John's average speed in miles per hour for the round trip: A to B and back to A? Round your answer to the nearest 0.1 miles per hour.

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5.2 (not 5.5)

Difficulty: challenging. The reason for listing the benchmark 7-I.1 is that a natural error which one can make in this problem gives an answer that is twice the correct answer, but a student should be aware that an answer larger than 6.7 cannot possibly be correct.

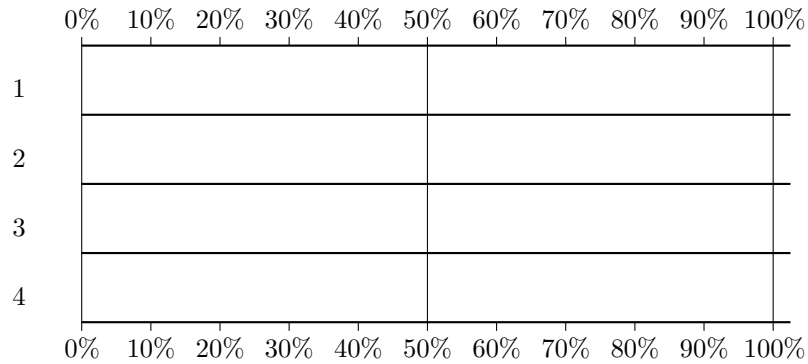
FOR GRADE 7, with comments and ANSWERS

7-II.A.1
6-IV.B.1
6-IV.A.1

A 4-faced die with the faces labeled as 1, 2, 3, and 4 is rolled 5128 times. [It is not known whether the die is well-balanced.] The results are:

- 1 occurred 1013 times
- 2 occurred 1380 times
- 3 occurred 1502 times
- 4 occurred the other times

Make an accurate bar graph showing the percentages of times that each of the four numbers occurred.



The relevant percentages are

20, 27, 29 24

for 1, 2, 3, and 4. I won't actually draw the bar graph.

Difficulty: standard-e. It is often misleading to compare percentages on a bar graph. Here, however, it is fine since all the percentages were obtained by dividing by the same denominator.

FOR GRADE 7, with comments and ANSWERS

7-II.B.3
7-II.B.7
7-I.3
7-I.4
7-I.1
7-II.B.7

Sometime ago a merchant set the price of a certain object at \$75.95. He has not been able to sell it so he has decided he would like to lower the price and advertise a sale. However, he is only willing to lower the price to \$69.50. He has decided he would like to advertise this price as 30% off the regular price. To do so, he must first raise the regular price from the original \$75.95 and, by law, keep it at that price for 21 days in order to advertise that price as the regular price. By what percentage should the merchant raise his price in order to accomplish his goal of a subsequent reduction by 30% percent to \$69.50? Round your answer up to the nearest tenth of a percent.

Price to which the merchant needs to raise is $69.50/.7 = 99.29$, which means he must raise the price 23.34 from its original price of 75.95.

$$\frac{23.34}{75.95} = 0.3073.$$

Thus the answer is 30.8%. [The reason for rounding up is to make sure that the subsequent discount is over 30% rather than under 30%, so that the sale is at least as good as advertised.]

Difficulty: challenging.

The views and opinions expressed in this link are strictly those of Bert Fristedt. The contents have been neither reviewed nor approved by the University of Minnesota.