PROBLEMS BASED ON THE SPRING 2003
MATHEMATICS STANDARDS AND BENCHMARKS

FOR GRADE 8

WITHOUT EXTRA COMMENTARY

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April 21, 2005

The purpose of this link from my web-site is to identify a selection of problems aligned with the Minnesota mathematics standards and benchmarks for Grade 8 as adopted in Spring 2003. My focus consists of the standards and benchmarks themselves; the problems here serve to illuminate them. The benchmarks and standards that are particularly relevant for a particular problem are identified in the left-hand margins; for instance, 8-II.B.4 indicates the Grade-8 benchmark II.B.4 and 8-II.B refers to the corresponding standard. In another sense, the focus is the suitability of problems for the Minnesota Comprehensive Assessments (know as MCA's), but in saying this I want to emphasize that the opinions are mine alone, formed without consultation with Minnesota Department of Education. Relevant to the above concerns is a companion link which includes the problems from this link, along with a variety of comments.

I was one of approximately 40 members of the mathematics subcommittee of the Academic Standards Committee, formed by the Minnesota Commissioner of Education in February 2003. I strongly support the mathematics standards and benchmarks resulting from the work of that committee and which, on the basis of a law passed by the Legislature and signed by the Governor, became official in Spring 2003. Although there is no guarantee that this web-site item reflects the thinking within the Department of Education, I have tried very hard to reflect the standards and benchmarks accurately, taking care not to bend them in the direction of my individual views. [Even though I strongly support the standards and benchmarks document, there are places where I would have preferred the document to be a bit different, and I suspect that the same is true (but not for the same places) of every member of the mathematics subcommittee.]

Anticipating that I might want to modify this document from time to time, I have refrained from labeling the problems with numerals and am planning to change the date at the top any time I make additions or changes.

Since the standards are cumulative, all the K-8 benchmarks are relevant for the Grade-8 MCA. It seems to me that it is desirable for Grade-8 teachers to examine all the K-8 benchmarks giving special attention to those for grades 6-8, and in general for teachers to read the standards for a couple grades on either side of the grade they are teaching.

The variety of different problems that are consistent with the standards and benchmarks is very large—that is the power of mathematics; a manageable number of basic principles and techniques enables one to handle a myriad of different situations. So, of course, the problem list that follows cannot be viewed

as comprehensive.

For problems in which students are to place the correct digits in boxes, a decimal point or comma is included between appropriate pairs of boxes when relevant. If the answer requires fewer digits than boxes, it is the left-hand box or boxes which should be left blank. [If the Grade-8 MCA were, in fact, to include such problems it would be important that students become familiar with the instructions some days in advance of the test.]

There is not a sharp demarcation separating problems appropriate for various grade levels. For instance, some of the problems described below for Grade 8 are also in the link for Grade 7. Typically, a problem that is appropriate for both the Grade-8 MCA and the Grade-7 MCA would be regarded as a more difficult problem for a seventh grader than it would be regarded for eighth graders.

It is clear from the benchmarks 8-II.B.1, 8-II.B.5, and 8-II.B.6, as well as some Grade-6 and Grade-7 benchmarks that the Grade-8 MCA should contain a significant section where a calculator is permitted. It is also clear from standard 8-II.B itself and benchmark 8-II.A.1, in combination with benchmarks from earlier grades, that there are a wide variety of problems which the student should be able to handle by hand.

The first part of the list below is relevant for the non-calculator portion of the Grade-8 MCA, and later, an introductory sentence identifies the place where the 'calculator permitted' portion begins.

I want to again emphasize: Although the standards and the benchmarks accompanying them constitute an official document of the state of Minnesota, all the judgments about alignment of problems with the benchmarks and standards are mine; neither do they have any official standing nor have they been obtained in consultation with the Minnesota Department of Education. Also, they have not been reviewed by the University of Minnesota where I am a faculty member and, of course, they do not represent any official view of that institution.

7-II.A.1 $62.5\% =$

(a) 5/8

(b) 5/6

(c) 25/4

(d) 125/2

FOR GRADE 8, WITHOUT EXTRA COMMENTARY

7-II.B.5 $5^3 =$

(a) 75
(b) 125
(c) 225
(d) 243

7-V.C.1 Which of the following is the most appropriate physical unit with which to describe the capacity of the gasoline tank in an automobile?

(a) centimeter
(b) kiloliter
(c) liter
(d) meter

7-III.A.2
8-III.A.2 Find the slope of the line in a plane that passes through the points $(4, 3)$ and $(-1, 1)$ with, as usual, the axis for first coordinates being the horizontal axis.

(a) 2/5
(b) 3/4
(c) 4/3
(d) 5/2

8-V.B.2 On a drawing of the floor plan of a particular house, one foot in the house is represented by 1/8 of an inch. How long is the actual house if the length as measured on the drawing is $4\frac{3}{4}$ inches?

(a) 17 feet 6 inches
(b) 19 feet
(c) 35 feet
(d) 38 feet

8-IV.B.2
8-I.3 If the odds against a certain event happening are 4 to 3, then the probability that this event will occur is

(a) 3/7
(b) 4/7
(c) 3/4
(d) 4/3

7-II.B.4 What does 5% of 2% equal?

(a) 0.1%
(b) 0.4%
(c) 10%
(d) 40%

8-V.B.5
8-II.B.7 A square and equilateral triangle each of side length 4 inches are sewn together along one side of each of these two shapes. What is the perimeter of the shape obtained by this sewing?

(a) 20 inches
(b) 24 inches
(c) 28 inches
(d) 96 inches

7-IV.B1
6-II.A.2
7-II.A.1 To the nearest whole percent, what is the probability, in percentages, of rolling a three with a perfectly balanced die?

(a) 6.7
(b) 16
(c) 16.7
(d) 17

8-II.A.2
8-III.B.4 Simplify $(5\sqrt{3}+\sqrt{2})^2$.

(a) 17
(b) 79
(c) $79+5\sqrt{6}$
(d) $79+10\sqrt{6}$

8-V.B.4
8-III.B.4 Let h and r denote the height and radius, respectively, of a right circular cylinder closed on both the bottom and the top. Which of the following is a formula for its surface area?

(a) $\pi r^2 h$
(b) $\pi(r^2+rh)$
(c) $\pi(r^2+2rh)$
(d) $\pi(2r^2+2rh)$

FOR GRADE 8, WITHOUT EXTRA COMMENTARY

8-V.B.3
8-III.B.3

A ball made out of a certain material weighs 32 pounds and has a diameter of 6 inches. Another ball of the same material weighs 4 pounds. What is its diameter in inches?

(a) 3/4
(b) $3/\sqrt{2}$
(c) 3/2
(d) 3

8-II.A.1
8-II.B.4

Which of the following statements is true?

(a) $\sqrt{2} < \frac{41}{29} < \frac{99}{70}$
(b) $\sqrt{2} < \frac{99}{70} < \frac{41}{29}$
(c) $\frac{41}{29} < \sqrt{2} < \frac{99}{70}$
(d) $\frac{41}{29} < \frac{99}{70} < \sqrt{2}$

8-V.B.3
8-I.3
5-I.2

The volume of the region between two concentric cylinders of height 3 and radii 2 and 5 equals $k\pi$ for some constant k. Find k.

(a) 9
(b) 27
(c) 63
(d) 189

8-III.B.1
8-III.B.4

Simplify the expression $x^2(x^3 - 5)$.

(a) $x^5 - 5x^2$
(b) $x^5 - 5$
(c) $x^6 - 5x^2$
(d) $x^6 - 5$

5-II.A.3
7-II.B.1

Take $\frac{11}{128}$ from $\frac{75}{128}$ and reduce the answer to lowest terms. Write the numerator in the left-hand boxes and the denominator in the right-hand boxes.

FOR GRADE 8, WITHOUT EXTRA COMMENTARY

8-III.A.1 The first, second, and third terms of a particular arithmetic progression are 7, 18, and 29, in that order. Find the fifth term.

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8-III.B.3 **8-II.A.1** What percentage of the following numbers are greater than $^{-}3\frac{2}{5}$:

$$^{-}7\,,\quad 7\,,\quad ^{-}7/2\,,\quad 7/2\,,\quad ^{-}3.72\,,\quad 3.72\,,\quad ^{-}3\,,\quad 3\,,\quad ^{-}2/7\,,\quad 2/7\,?$$

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7-II.B.1 **7-II.B.8** **6-II.B.2** Calculate

$$\frac{4}{21}-\frac{2}{35}\,,$$

writing your answer in lowest terms. Enter the numerator in the left-hand boxes and the denominator in the right-hand boxes.

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7-III.B.3 **7-III.B.1** **7-II.B.5** Suppose x, y, and z are related by the formula $z = xy^2$. Find x when $y = 3$ and $z = 117$.

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7-II.B.4 **7-II.B.1** **7-II.B.8** Calculate

$$5\,\frac{5}{6}\times 2\,\frac{4}{5}\,.$$

Write the answer as a mixed number with the fractional part in lowest terms. Then place the numerator of the fractional part of the answer in the left-hand boxes and the denominator of the fractional part in the right-hand boxes.

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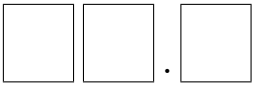
8-II.A.3 The quotient $(4 \times 10^7) \div (5 \times 10^{(} - 2)$ can be written as a one-digit integer times a power of 10. Write the one-digit integer in the left-hand box and the power of 10 in the right-hand boxes.

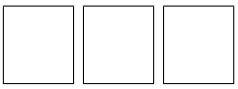
7-II.A.1
8-II.A.4 Write $3.4\overline{87}$ as a fraction in lowest terms. [Note: the bar above 87 indicates that the decimal numeral goes on forever with the repeating pattern 87.] Enter the numerator in the left-hand boxes and the denominator in the right-hand boxes.

8-I.3
6-II.B.4
8-I.1 Points A and B are 25.2 miles apart. John bicycles from A to B at an average speed of 8.4 miles per hour and returns from B to A at an average speed of 6.3 miles per hour. What is John's average speed in miles per hour for the round trip: A to B and back to A? Round your answer to the nearest 0.1 miles per hour.



8-V.B.4
7-III.B.3
8-I.3
4-V.B.1
5-I.2

The formula for the surface area of a rectangular parallelepiped in which two faces are squares is $2(x^2 + 2xy)$, where x is the edge length of the square faces and y is the length of the other edges of the parallelepiped. Find y when the surface area equals 410 and $x = 5$. *Hint:* The answer is a whole number.



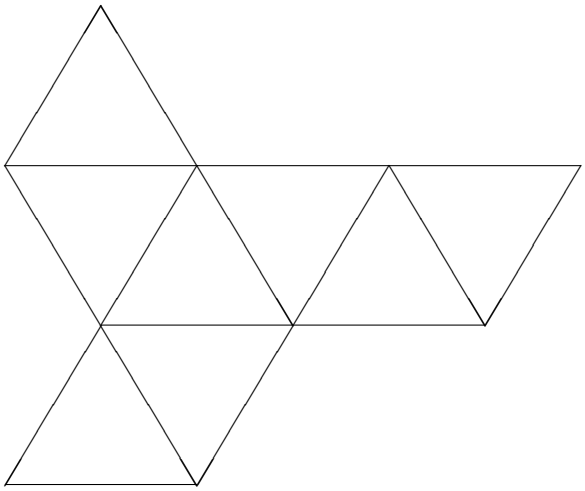
8-II.B.4
7-II.A.1
7-II.B.1

The cube root of

$$\frac{63}{149\frac{1}{3}}$$

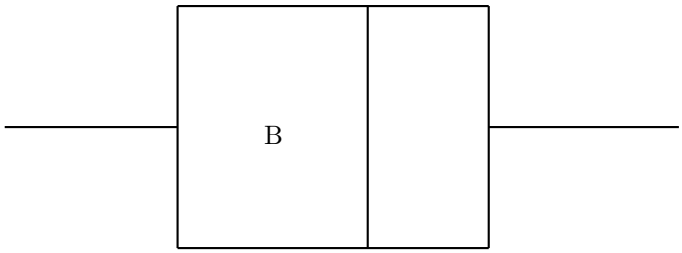
is a rational number. Represent this cube root as a fraction in lowest terms, using the left-hand boxes for the numerator and the right-hand boxes for the denominator.

8-V.A.1 The following pattern can be folded along the lines into an octahedron. At each vertex on the paper place one of the labels A, B, C, D, E, or F in such a way the all vertices with the same label become the same vertex when it is folded into an octahedron and those with different labels on the paper remain different after folding. Recall that an octahedron has eight triangular faces and six vertices, with four edges meeting at each vertex. *Hint:* Three labels should each be used once, two labels should each be used twice, and one label should be used thrice.



8-V.B.2 The side lengths of a certain triangle are 3, 5, and $6\frac{1}{3}$. The shortest side in a similar triangle has length $4\frac{1}{2}$. Find the length of the longest side of this similar triangle.

8-IV.A.2 For a box-and-whiskers diagram such as the one below describe the interpretation of the box in which the letter B is written:



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6-II.B.1 Calculate the prime factorization of $6,438,000$. Show your work and make your reasoning clear.

7-IV.B.2
8-I.4 A die is rolled three times in succession, and each of the three times a five is obtained. Some people might say that the probability of obtaining a five on the next role equals $1/6$, whereas others might say that this probability is significantly larger than $1/6$. Depending on one's point of view, one can support either of these assertions. Explain.

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8-V.C Here are two facts about liquid measure:
8-I.4 (i) one gill equals one-fourth of a pint;
8-I.3 (ii) one U.S. barrel equals $31\frac{1}{2}$ gallons.
7-II.B.1 How many gills are there in 2/3 of a barrel?

8-IV.A.2 Consider 29 distinct data of real numbers. Counting from the smallest datum
8-I.2 upwards, at what count does one arrive at the median? At what count does
8-I.4 one arrive at the third quartile? Generalize your two answers to the situation
8-III.B.4 in which there are $4n+1$ distinct data of real numbers for some positive integer n.

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The following problems are designed with the **calculator portion of the Grade-8 MCA** in mind.

8-II.B.5
8-I.1

With accuracy in two places to the right of the decimal point,

$$7.77 \times [3329.23 - (45872.1 - 955.25)]$$

equals

(a) $-337,980.39$
(b) $-323,135.81$
(c) $-20,959.23$
(d) $-19,048.71$

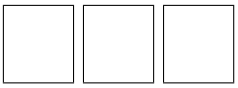
7-II.B.5
8-II.B.5
8-I.1
7-II.B.8

Calculate 3×5^6. *Reminder:* The four options below are sufficiently different that one can actually avoid using a calculator by instead making an estimate.

(a) $23,328$
(b) $46,875$
(c) $1,889,568$
(d) $11,390,625$

6-V.B.3
6-V.C.1

The circumference of a certain circle equals 7 feet, 5 inches. Calculate its radius to the nearest inch. You may use the approximation 3.14 for π or you may use the key for π itself on your calculator.



7-II.B.2
6-V.C.3
8-I.4
8-I.1

To three decimal-place accuracy find the edge length in inches of a square whose area equals that of a rectangle whose edge length measurements are 5 inches and 12 inches.

7-II.B.3
8-I.3
8-I.1

To what amount does \$3050 grow after 4 years 8 months at an annual simple interest rate of $5\frac{1}{4}$ per cent. Round your answer to nearest cent.

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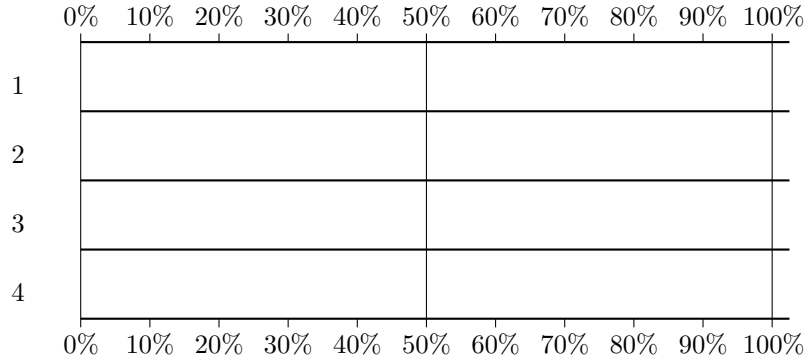
7-II.A.1
6-IV.B.1
6-IV.A.1

A 4-faced die with the faces labeled as 1, 2, 3, and 4 is rolled 5128 times. [It is not known whether the die is well-balanced.] The results are:

1 occurred 1013 times
2 occurred 1380 times
3 occurred 1502 times
4 occurred the other times

Make an accurate bar graph showing the percentages of times that each of the four numbers occurred.



FOR GRADE 8, WITHOUT EXTRA COMMENTARY

8-V.C.1 There are 2.2046223 pounds in one kilogram, correct to eight significant figures. To six significant figures, how many grams are in one pound? Show your work and reasoning.

8-III.A.3
8-II.B.4 Suppose that y and x are related by the formula $y = x^2 + \sqrt{x}$. For six equally-spaced values of x beginning with $x = 0$ and ending with $x = 2$, calculate the corresponding approximate values of y to three decimal places, arrange the results of your calculations in a table, and sketch a graph showing the relation between x and y.

7-II.B.3
7-II.B.7
8-I.3
8-I.4
8-I.1
7-II.B.7

Sometime ago a merchant set the price of a certain object at \$75.95. He has not been able to sell it so he has decided he would like to lower the price and advertise a sale. However, he is only willing to lower the price to \$69.50. He has decided he would like to advertise this price as 30% off the regular price. To do so, he must first raise the regular price from the original \$75.95 and, by law, keep it at that price for 21 days in order to advertise that price as the regular price. By what percentage should the merchant raise his price in order to accomplish his goal of a subsequent reduction by 30% percent to \$69.50? Round your answer up to the nearest tenth of a percent.