

(April 7, 2011)

Modular forms and number theory exercises 16

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[mfms 16.1] Some calculus: derive the ordinary differential equation

$$\Delta f(|x|) = f''(|x|) + \frac{n-1}{|x|} f'(|x|)$$

[mfms 16.2] More calculus: eliminate the first-derivative term in a differential equation

$$f'' + pf' + qf = 0$$

by letting $f = u \cdot w$, expanding, setting the u' coefficient to 0, and solving the resulting differential equation for w . Do this for the differential equation above.

[mfms 16.3] Execute the previous normalization procedure, and then irregular singular point discussion, to see that the leading term in the asymptotics for solutions of

$$f''(|x|) + \frac{n-1}{|x|} f'(|x|) - \lambda f = 0$$

really is

$$f(|x|) \sim \frac{e^{\pm\sqrt{\lambda}|x|}}{|x|^{\frac{n-2}{2}}}$$

[mfms 16.4]* Carry out the same calculus exercises for $f(x) = P(x)u(|x|)$, where P is a homogeneous, harmonic, degree d polynomial.