

Math 2243  
Fall 2006  
Midterm 2  
October 24, 2006  
Time Limit: 50 minutes

Name (Print): \_\_\_\_\_  
Student ID: \_\_\_\_\_  
Section Number: \_\_\_\_\_  
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This exams contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (single - sided) 8.5 inch  $\times$  11 inch sheet of notes into the exams.

Do not give numerical approximations to quantities such as  $\sin 5$ ,  $\pi$ , or  $\sqrt{2}$ . However, you should simplify  $\cos \frac{\pi}{2} = 0$ ,  $e^0 = 1$ , and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including a brief justification of the evaluation of definite and indefinite integrals.**
- **Mysterious or unsupported answers will not receive full credit.** Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit;** an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive very little credit.
- You may use a **crib sheet** which you have prepared in advance. The crib sheet may only be a half page ( $8\frac{1}{2} \times 5\frac{1}{2}$  inches), on one side.
- No **calculators** are allowed, nor will they be needed.

1	15 pts	
2	25 pts	
3	15 pts	
4	10 pts	
5	10 pts	
6	25 pts	
TOTAL	100 pts	

1. (15 points) Find all solutions of this system of three linear equations in five unknowns  $x, y, x, u, v$ .  
(**Hint:** the system is in reduced row-echelon form already!)

$$x + 2z + v = 0$$

$$y - 3z = 2$$

$$u + 5v = 4.$$

2. (25 points) Find all solutions to this system of three linear equations in three unknowns  $(x, y, z)$ :

$$\begin{aligned}x + y + z &= 5 \\2x - 3y + 7z &= 0 \\4x - y + 9z &= 10.\end{aligned}$$

Show your work! (Partial credit for row-echelon form.)

3. (15 points) (True or False)  $\mathcal{C}(-\infty, \infty)$  is the vector space of all continuous functions on the real line  $(-\infty, \infty)$ . Which of the following sets are **subspaces** of  $\mathcal{C}(-\infty, \infty)$ ?

(a) The set of functions  $y(t)$  with  $y(0) = 0$  and  $\frac{dy}{dt}(0) = 0$ .

SUBSPACE	
NOT A SUBSPACE	

(b) The set of functions  $y(t)$  with the product  $y(1)y(-1) = 0$ .

SUBSPACE	
NOT A SUBSPACE	

(c) The set of solutions  $y(t)$  of the differential equation

$$\frac{dy}{dt} + (y(t))^2 = 0.$$

SUBSPACE	
NOT A SUBSPACE	

4. (10 points) Determine whether or not the three vectors  $\vec{u}$ ,  $\vec{v}$  and  $\vec{w}$  in  $\mathbb{R}^3$  form a **basis** for  $\mathbb{R}^3$ . Why or why not?

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}.$$

5. (10 points) The  $3 \times 3$  matrix  $A$  has one entry  $a_{23} = t$  which depends on  $t$ , so the determinant of  $A$  is a function of  $t$ . Find its derivative

$$\frac{d}{dt} \begin{vmatrix} 2 & 4 & 9 \\ -1 & -2 & t \\ 3 & 5 & 7 \end{vmatrix}.$$

(**Hint:** try expanding the determinant in the second row).

6. (25 points) Consider the system of differential equations:

$$\begin{aligned}\frac{dx}{dt}(t) &= 2x(t) + y(t) \\ \frac{dy}{dt}(t) &= x(t) - 2y(t).\end{aligned}$$

- (a) Sketch the  $v$ -nullclines and the  $h$ -nullclines in the  $(x, y)$ -plane.

- (b) On the same sketch, draw a few arrows on and off the nullclines, indicating where  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$  are positive or negative.

- (c) Is the equilibrium point  $(0, 0)$  **stable** or **unstable**?