Math 2243	**Name (Print):** ____________________
Fall 2006	**Student ID:** ____________________
Midterm 3	**Section Number:** __________
November 30, 2006	**Teaching Assistant:** ____________________
Time Limit: 50 minutes	**Signature:** ____________________

This exams contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (single - sided) 8.5 inch $\times$ 11 inch sheet of notes into the exams.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{2} = 0$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including a brief justification of the evaluation of definite and indefinite integrals**.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive very little credit.
- You may use a **crib sheet** which you have prepared in advance. The crib sheet may only be a half page ($8\frac{1}{2} \times 5\frac{1}{2}$ inches), on one side.
- No **calculators** are allowed, nor will they be needed.

1	20 pts	
2	15 pts	
3	20 pts	
4	25 pts	
5	20 pts	
TOTAL	100 pts	

1. (20 points) The matrix

$$A = \begin{bmatrix} -5 & -4 \\ 12 & 9 \end{bmatrix}$$

has eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 3$. Find corresponding **eigenvectors** $\vec{v}_1$ and $\vec{v}_2$.

2. (15 points) Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined by $T(\vec{v}) = A\vec{v}$, where the 3×3 matrix A has eigenvalues $\lambda_1 = 2$, $\lambda_2 = 0$, and $\lambda_3 = -1$; with corresponding eigenvectors $\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\vec{v_3} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$.

(a) (4 points) What is the **kernel** (or nullspace) of T?

(b) (4 points) Find the **rank** of T.

(c) (7 points) What subspace of $\mathbb{R}^3$ is the **image** of T? Describe the image of T in the form

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : ax_1 + bx_2 + cx_3 = 0 \right\}.$$

3. (20 points) Find a **particular solution** $y_p(t)$ for the nonhomogeneous differential equation

$$y''(t) - 4y'(t) = 9te^t.$$

4. (25 points) The matrix

$$A = \begin{bmatrix} 5 & 3 & 6 \\ -10 & -8 & -10 \\ 4 & 4 & 5 \end{bmatrix}$$

has eigenvalues $\lambda_1 = -1$, $\lambda_2 = 2$, and $\lambda_3 = -3$; with corresponding eigenvectors $\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{v_3} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$. Consider the nonhomogeneous system of first-order DEs

$$\vec{x}'(t) = A\vec{x}(t) + \vec{b},$$

where $\vec{b} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$. A particular solution is $\vec{x_p}(t) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ (a constant solution).

(a) (15 points) Find the **general solution**.

(b) (10 points) Find the solution which satisfies the **initial condition** $\vec{x}(0) = \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix}$.

5. (20 points) The 2×2 matrix

$$A = \begin{bmatrix} 1 & 9 \\ -1 & -5 \end{bmatrix}$$

has a double eigenvalue $\lambda_1 = \lambda_2 = -2$ and only one independent eigenvector $\vec{v} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

(a) (10 points) Find a **generalized eigenvector**.

(b) (10 points) Find two independent solutions of the system of DEs

$$\vec{x}'(t) = A\vec{x}(t).$$