

Math 2263  
Fall 2009  
Midterm 1  
October 8, 2009  
Time Limit: 50 minutes

Name (Print): \_\_\_\_\_  
Student ID: \_\_\_\_\_  
Section Number: \_\_\_\_\_  
Teaching Assistant: \_\_\_\_\_  
Signature: \_\_\_\_\_

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This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. Calculators may be used. Please turn off cell phones. You are allowed to bring one-half of one single-sided 8.5 inch  $\times$  11 inch page of notes, in your own handwriting, to the exam.

Do not give numerical approximations to quantities such as  $\sin 5$ ,  $\pi$ , or  $\sqrt{2}$ . However, you should simplify  $\cos \frac{\pi}{2} = 0$ ,  $e^0 = 1$ , and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit.** Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit;** an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

1	15 pts	
2	13 pts	
3	15 pts	
4	15 pts	
5	12 pts	
6	15 pts	
7	15 pts	
TOTAL	100 pts	

1. (15 points) The lines given parametrically by

$$(x, y, z) = (1 + t, -2 - 2t, 2t), \quad -\infty < t < \infty$$

and

$$(x, y, z) = (1 + 2s, -2 + 2s, s), \quad -\infty < s < \infty$$

intersect at the point  $(x, y, z) = (1, -2, 0)$ . Find an equation for the **plane which contains both lines**. Write the equation in the form  $ax + by + cz = d$ .

2. (13 points) Find an equation for the **hyperbolic cylinder** in  $(x, y, z)$ -space containing infinitely many lines parallel to the  $x$ -axis, and containing the slanted hyperbola

$$x = y, \quad x^2 + y^2 - z^2 = 4.$$

3. (15 points) Evaluate the **limit**

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - xy^3 + y^4}{x^4 + x^2y^2 + y^4},$$

or state that it does not exist, giving reasons.

4. (15 points) For the function  $f(x, y) = (x^2 - y^2)e^{xy}$ , find the **second partial derivatives** at  $x = 1, y = -1$ :

$$f_{xx}(1, -1), \quad f_{xy}(1, -1) \quad \text{and} \quad f_{yy}(1, -1).$$

5. (12 points) Suppose  $z = f(x, y)$  is a function with first partial derivatives  $f_x(1, 3) = -2$  and  $f_y(1, 3) = -1$ . If  $x$  and  $y$  are both functions of  $t$ :  $x = g(t) = -3 + 4t$  and  $y = h(t) = 6 - 3t$ , find the **derivative at  $t = 1$** :

$$\frac{dz}{dt}(1) = \frac{d}{dt}f(g(t), h(t))(1).$$

6. (15 points) A new set of coordinates  $(r, \theta)$  (polar coordinates) are defined by  $x = r \cos \theta$  and  $y = r \sin \theta$ . The partial derivatives of  $r$  and  $\theta$  are

$$r_x = \cos \theta, \quad r_y = \sin \theta,$$
$$\theta_x = -\frac{\sin \theta}{r}, \quad \theta_y = \frac{\cos \theta}{r}.$$

If  $f(x, y)$  is a continuously differentiable function and  $r \neq 0$ , compute a formula for

$$|\vec{\nabla} f|^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

in terms of  $r, \theta$  and the partial derivatives of  $f$  with respect to  $r$  and  $\theta$ . Simplify!

7. **(15 points)** (a) (7 of 15 points) Find the **gradient** of the function

$$f(x, y, z) = (x + z^2) \sin(xy)$$

at the point  $(x, y, z) = (1, \frac{\pi}{2}, -2)$ .

- (15 points)** (b) (8 of 15 points) Find the **directional derivative**  $D_{\vec{u}}f$  of  $f$  at the point  $(1, \frac{\pi}{2}, -2)$  in the direction of the unit vector

$$\vec{u} = \frac{1}{3}(2\vec{i} + \vec{j} + 2\vec{k}).$$