Math 2263	**Name (Print):** ____________________
Fall 2008	**Student ID:** ____________________
Midterm 2	**Section Number:** __________
October 30, 2008	**Teaching Assistant:** ____________________
Time Limit: 50 minutes	**Signature:** ____________________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. **Calculators may be used.** Please turn off cell phones. **Crib sheet:** You are allowed to have **one-half** of one single - sided 8.5 inch $\times$ 11 inch sheet of notes in the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{2} = 0$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

1	10 pts	
2	10 pts	
3	10 pts	
4	15 pts	
5	10 pts	
6	20 pts	
6	25 pts	
TOTAL	100 pts	

1. (10 points) Let R be the rectangle $-2 \leq x \leq 2$, $0 \leq y \leq 1$. Find the double integral

$$\iint_R \frac{x^2}{y+1}\, dA.$$

2. (10 points) Let R be the rectangle $0 \le x \le 5$, $0 \le y \le 2$ in the (x, y)-plane. If a continuous function $f(x, y)$ satisfies

$$-1 \le f(x, y) \le 3 - y^2,$$

what does this tell you about the value of $\iint_R f(x, y)\, dA$?

3. (10 points) Let D be the region $0 \leq x \leq 2$, $0 \leq y \leq x^2$ in the (x, y)-plane. Find the double integral

$$\iint_D xy\, dA.$$

4. (15 points) A plate is in the shape of the triangle D: $0 \leq y \leq 1 - |x|$, with corners $(-1, 0)$, $(0, 1)$ and $(1, 0)$. The plate has mass density at the point (x, y) equal to $\rho(x, y) = y$ per unit area.

   (a) (5 points) Find the **total mass** m of the plate.

   (b) (10 points) Find the **center of mass** $(\overline{x}, \overline{y})$ of the plate.

5. **(10 points) (a)** (5 points) Let D be the circular disk of radius R and center $(0,0)$ in the (x,y)-plane. Find

$$\iint_D \sin(x^2+y^2)\,dA.$$

(*Hint:* polar coordinates.)

6. (15 points) Find the **maximum** and **minimum** values of

$$f(x,y) = xy - x$$

subject to the **constraint**

$$g(x,y) = x^2 + 4y^2 = 4.$$

(*Hint:* Lagrange multipliers.)

7. (25 points) Let $f(x,y) = 3y^3 - x^2y + x^2 + \frac{9}{2}y^2$.

(a) (5 points) Compute the **first** and **second** partial derivatives of $f(x,y)$.

(b) (10 points) Find all the **critical points** of $f(x,y)$.

(c) (10 points) For each critical point, state whether it is a **local minimum point**, a **local maximum point** or a **saddle point**.