Math 2263
Spring 2008
Midterm 1
February 21, 2008
Time Limit: 50 minutes

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\begin{aligned}
\text { Name (Print): } & \\
\text { Student ID: } & \square \\
\text { Section Number: } & \square \\
\text { Teaching Assistant: } & \square \\
\text { Signature: } &
\end{aligned}
$$

This exams contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. Calculators may be used. Please turn off cell phones.
Do not give numerical approximations to quantities such as $\sin 5$, $\pi$, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{2}=0, e^{0}=1$, and so on.

The following rules apply:

- Show your work, in a reasonably neat and coherent way, in the space provided. All answers must be justified by valid mathematical reasoning. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- Mysterious or unsupported answers will not receive full credit. Your work should be mathematically correct and carefully and legibly written.
- A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

| 1 | 15 pts |  |
| :---: | :---: | :--- |
| 2 | 15 pts |  |
| 3 | 15 pts |  |
| 4 | 15 pts |  |
| 5 | 10 pts |  |
| 6 | 15 pts |  |
| 7 | 15 pts |  |
| TOTAL | 100 pts |  |

1. (15 points) Find an equation for the plane passing through all three points $\langle x, y, z\rangle=\langle 3,-2,-2\rangle,\langle 2,0,1\rangle$ and $\langle 1,0,0\rangle$.
2. (15 points) Find an equation for the surface in ( $x, y, z$ )-space obtained by rotating the hyperbola $x^{2}-4 z^{2}=1$ of the $(x, z)$-plane about the $x$-axis.
3. (15 points) The lines given paramerically by

$$
(x, y, z)=(7+2 t,-1-t,-2 t), \quad-\infty<t<\infty
$$

and

$$
(x, y, z)=(4-s,-1+2 s, 2+2 s), \quad-\infty<s<\infty
$$

intersect at the point $\langle x, y, z\rangle=\langle 3,1,4\rangle$. Find an equation for the plane which contains both lines.
4. (15 points) For the function $f(x, y)=e^{-2 y} \sin 2 x$, find the second partial derivatives

$$
f_{x x}=\frac{\partial^{2} f}{\partial x^{2}}
$$

and

$$
f_{y y}=\frac{\partial^{2} f}{\partial y^{2}}
$$

5. (10 points) Suppose $z=f(x, y)$ is a function with partial derivatives $f_{x}(3,1)=5$ and $f_{y}(3,1)=$ 2. If $x$ and $y$ are both functions of $t: x=5-2 t$ and $y=2+t-2 t^{2}$, find

$$
\frac{d z}{d t}
$$

at $t=1$.
6. (15 points) The point $\langle x, y, z\rangle=\langle 2,1,0\rangle$ lies on the surface $S$ :

$$
x^{2}-y^{2}+x z+x y-4 z^{2}=5 .
$$

Find the equation of the tangent plane to the surface $S$ at $\langle 2,1,0\rangle$, in the form $a x+b y+c z=d$.
7. (15 points) (a)Find the gradient of the function $f(x, y, z)=e^{z} \ln (x+2 y)$ at the point $\langle x, y, z\rangle=\langle e, 0,1\rangle$. (b) Find the directional derivative of $f$ at the point $\langle e, 0,1\rangle$ in the direction

$$
\vec{u}=\frac{1}{3}(\vec{i}-2 \vec{j}+2 \vec{k}) .
$$

