Math 2263
Spring 2008
Midterm 3
April 24, 2008
Time Limit: 50 minutes

Name (Print): ____________________
Student ID: ____________________
Section Number: __________
Teaching Assistant: ____________________
Signature: ____________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. **Calculators may be used.** Please turn off cell phones.
Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos\frac{\pi}{2} = 0$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

1	10 pts	
2	20 pts	
3	15 pts	
4	20 pts	
5	20 pts	
6	15 pts	
TOTAL	100 pts	

1. (10 points) Let B be the rectangular solid $0 \leq x \leq 4$, $1 \leq y \leq 2$, $0 \leq z \leq 2$. Find

$$\iiint_B \frac{xz^2}{y^2}\, dV.$$

2. (20 points) Let $\vec{F}$ be the vector field

$$\vec{F}(x, y) = (y^2 + e^x)\,\vec{i} + 2xy\vec{j}.$$

(a) (10 points) Find a real-valued function $f(x, y)$ so that

$$\nabla f(x, y) = \vec{F}(x, y).$$

(b) (10 points) Let C be the curve given by $x = \cos(t^2\pi)$ and $y = t$, $0 \leq t \leq 2$. Find the line integral

$$\int_C \vec{F} \cdot d\vec{r}.$$

3. (15 points) Find the **surface area** of the portion of the paraboloid $z = x^2 + y^2$ which lies below the plane $z = 2$ (*Hint:* you might want to compute the double integral in polar coordinates.)

4. (20 points) Under the linear transformation $x = 4u + v$, $y = 5u + 3v$ from the (u, v)-plane to the (x, y)-plane, the circular disk D given by the inequality $u^2 + v^2 \leq 4$ is transformed into the elliptical region E given by $34x^2 - 46xy + 17y^2 \leq 156$. Compute the **area of** E **as an integral over** D.

5. (20 points) An oriented curve C in the (x, y)-plane consists of four pieces C_1, C_2, C_3 and C_4:
C_1 is the upper semicircle from $(2, 0)$ to $(-2, 0)$, given by $x = 2\cos t,\ y = 2\sin t,\ 0 \leq t \leq \pi$; C_2 is the segment of the x-axis from $(-2, 0)$ to $(-1, 0)$;
C_3 is the upper semicircle from $(-1, 0)$ to $(1, 0)$, given by $x = -\cos t,\ y = \sin t,\ 0 \leq t \leq \pi$; and
C_4 is the segment of the x-axis from $(1, 0)$ to $(2, 0)$.
Let $\vec{F}$ be the vector field

$$\vec{F}(x, y) = [e^{x^2} + \sin y]\ \vec{i} + [x\cos y + 3x + \ln(y + 1)]\ \vec{j}.$$

Find $\int_C \vec{F} \cdot d\vec{r}$. (*Hint:* Green's Theorem makes this much easier! You may use what you remember about areas inside circles.)

6. (15 points) The portion of the ball of radius 2 with center at $(0,0,0)$, which lies above the cone
$$z = \frac{1}{2}\sqrt{x^2 + y^2 + z^2},$$
is described in spherical coordinates by $0 \le \rho \le 2$, $0 \le \phi \le \pi/3$, $0 \le \theta \le 2\pi$. Find the **volume** of this figure by computing an integral in spherical coordinates. (*Hint:* Recall that $\sin \pi/3 = \frac{1}{2}\sqrt{3}$ and $\cos \pi/3 = \frac{1}{2}$.)