Math 2263	**Name (Print):** ______________
Spring 2009	**Student ID:** ______________
Midterm 2	**Section Number:** ________
March 26, 2009	**Teaching Assistant:** ______________
Time Limit: 50 minutes	**Signature:** ______________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. **Calculators may be used.** Please turn off cell phones. **Crib sheet:** You are allowed to bring **one-half** of one single - sided 8.5 inch $\times$ 11 inch sheet of notes to the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos\frac{\pi}{2} = 0$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

1	10 pts	
2	15 pts	
3	10 pts	
4	15 pts	
5	10 pts	
6	25 pts	
7	15 pts	
TOTAL	100 pts	

1. (10 points) Let R be the rectangle $0 \leq x \leq 3$, $1 \leq y \leq 3$. Find the double integral

$$\iint_R \left(x^2 - 3xy\right) dA.$$

2. (15 points) Let R be the square $-2 \leq x \leq 2$, $-2 \leq y \leq 2$ in the (x, y)-plane. If $f(x, y)$ is a continuous function, and you only know that it satisfies

$$1 \leq f(x, y) \leq 5 - x + y,$$

what does this tell you about the value of $\iint_R f(x, y)\, dA$?

3. (10 points) Let D be the triangle $0 \leq x \leq \sqrt{\pi}$, $0 \leq y \leq x$ in the (x, y)-plane. Find the double integral

$$\iint_D \sin(x^2)\, dA.$$

4. (15 points) A plate is in the shape of the rectangle D: $-1 \leq y \leq 1$, $0 \leq x \leq 1$. The plate has mass density per unit area at the point (x, y) equal to $\rho(x, y) = 1 + x + y$.
   (a) (5 points) Find the **total mass** m of the plate.

   (b) (10 points) Find the **center of mass** $(\overline{x}, \overline{y})$ of the plate.

5. (10 points) Let D be the circular disk of radius R and center $(0,0)$ in the (x,y)-plane. Find

$$\iint_D e^{-(x^2+y^2)}\, dA$$

in terms of the radius R. (*Hint:* polar coordinates.)

6. (25 points) Let $f(x, y) = 3x^3 - xy^2 + \frac{9}{2}x^2 + y^2$.

(a) (5 points) Compute the **first** and **second** partial derivatives of $f(x, y)$.

(b) (10 points) Find all the **critical points** of $f(x, y)$.

(c) (10 points) For each critical point, state whether it is a **local minimum point**, a **local maximum point** or a **saddle point**.

7. (15 points) Find the **maximum** and **minimum** values of

$$f(x, y) = xy + 2y$$

on the circle of radius 2 centered at the origin. That is, the allowed points (x, y) are subject to the **constraint**

$$g(x, y) = x^2 + y^2 = 4.$$

(*Hint:* Lagrange multipliers.)