Math 2263	**Name (Print):** ____________________
Spring 2009	**Student ID:** ____________________
Midterm 3	**Section Number:** __________
April 23, 2009	**Teaching Assistant:** ____________________
Time Limit: 50 minutes	**Signature:** ____________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. **Calculators may be used.** Please turn off cell phones. You may refer to your crib sheet, a half page on one side.
Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{2} = 0$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.

- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.

- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

1	10 pts	
2	15 pts	
3	25 pts	
4	15 pts	
5	20 pts	
6	15 pts	
TOTAL	100 pts	

1. (10 points) Let B be the box, or rectangular solid: $0 \leq x \leq 2$, $0 \leq y \leq 3$, $0 \leq z \leq 1$. Find

$$\iiint_B (xy - 2yz + x^2 z^2)\, dV.$$

2. (15 points) Let E be the solid region bounded below by the cone $z^2 = x^2 + y^2$ and above by the plane $z = 1$. Find

$$\iiint_E (x^2 + y^{)}2 \, dV.$$

Hint: try cylindrical coordinates.

3. (25 points) C_1 and C_2 are oriented curves in the (x, y)-plane, each of which starts at $(0, 0)$ and ends at $(1, 1)$. C_1 is given by $y = x^2$, $0 \leq x \leq 1$; and C_2 is given by $x = y^2$, $0 \leq y \leq 1$. Let the vector field $\vec{F}$ be given by

$$\vec{F}(x, y) = 2xy\vec{i} + (x^2 - y^2)\vec{j}.$$

(a) (10 points) Find

$$\int_{C_1} \vec{F} \cdot d\vec{r}.$$

(b) (10 points) Find

$$\int_{C_2} \vec{F} \cdot d\vec{r}.$$

(c) (5 points) Is the vector field $\vec{F}(x, y)$ **conservative**? Why or why not?

4. (15 points) Under the transformation $x = 4u + v$, $y = 5u + 2v$ from the (u, v)-plane to the (x, y)-plane, the circular disk D given by the inequality $u^2 + v^2 \leq 9$ is transformed into the elliptical region E given by

$$85x^2 - 76xy + 17y^2 \leq 9.$$

Compute the **area of E as an integral over D**.

5. (20 points) Let C be the circle $x^2+y^2=9$ in the (x,y)-plane, oriented counterclockwise. Find

$$\oint_C (2xye^{x^2}+2y^2-y)\,dx+(e^{x^2}+4xy-3x)\,dy.$$

Hint: try Green's theorem!

6. (15 points) The portion E of the ball of radius 2 with center at $(0, 0, 0)$, which lies above the cone

$$z = \sqrt{x^2 + y^2},$$

is described in spherical coordinates by $0 \leq \rho \leq 2$, $0 \leq \phi \leq \pi/4$, $0 \leq \theta \leq 2\pi$. Find the **volume** of this figure by computing an integral in spherical coordinates. (*Hint:* Recall that $\sin \pi/2 = \frac{1}{2}\sqrt{2} = \cos \pi/2$.)