

Math 1272 Final Review Problems

1. Evaluate $\int_0^{2\sqrt{3}/5} \frac{x^2}{\sqrt{16-25x^2}} dx$.

2. Evaluate $\int \sin^2(4x) \cos^3(4x) dx$.

3. Evaluate $\int \frac{72x-20}{(3x^2+5)(x+6)} dx$.

4. Evaluate $\int x e^{4x} dx$

5. Find the arc length of the arc $y = (2/9)(3x+12)^{3/2}$ for $4 \leq x \leq 12$.

6. Explain why the following integral is improper. Evaluate it and state whether it converges or diverges.

$$\int_0^{\infty} \frac{x}{(16+x^2)^{3/2}} dx$$

7. The region bounded by the parabola $y = 12x - x^2$ and the parabola $y = x^2 - 2x$ is covered by a lamina of constant density ρ . Find M_x and M_y , the moments about the x -axis and y -axis, and also find the mass of the lamina. What is the center of mass of the lamina?

8. Solve the IVP $\frac{dy}{dx} = \frac{2(y+14)}{2x+3}$ and $y(1) = 3$. Write the solution for y in terms of x .

9. Solve the differential equation

$$x \frac{dy}{dx} + 2y = x e^{-5x^3}$$

10. Find the area of the surface generated when the function $y = \sqrt{3x+10}$ is revolved around the x -axis from $x = 0$ to $x = 6$.

11. State why the following integral is improper. Determine if it converges or diverges. If it converges, find its value.

$$\int_2^8 \frac{dx}{\sqrt{x-2}}$$

12. Determine if the lines $\vec{r}(t) = (5t-6)\vec{i} + (-3t+22)\vec{j}$ and $\vec{R}(s) = (4s+26)\vec{i} + (-2s+4)\vec{j}$ are parallel, intersecting or skew. If they intersect, find the intersection point.

13. Let \vec{v} denote the vector with initial point $(4, 1, -5)$ and terminal point $(7, 1, 0)$. Let \vec{w} denote the vector with initial point $(4, 1, -5)$ and terminal point $(8, -5, -3)$. Find the vector $\vec{v} \times \vec{w}$. Graph \vec{w} and \vec{v} as given, then graph them as position vectors at the origin. Finally, graph $\vec{v} \times \vec{w}$ as a position vector at the origin.

14. Let $\vec{v} = 6\vec{i} - 7\vec{j} + 3\vec{k}$ and $\vec{w} = 8\vec{i} + 6\vec{j} - 9\vec{k}$. Find the angle between \vec{v} and \vec{w} in radians correct to 4 decimal places. Find $\text{Proj}_{\vec{w}}\vec{v}$.

15. Find the vector function whose graph is the line through the two points $(5, 0, -4)$ and $(4, -5, -4)$.

16. Evaluate $\int x^2 \sin(3x) dx$.

17. Evaluate $\int \tan^3(4x) dx$.
18. For what values of x is the power series $\sum_{n=0}^{\infty} \frac{(-1)^n(4n+5)x^n}{4^n(n+1)}$ convergent? (Find the radius and interval of convergence).
19. Determine if the series $\sum_{n=1}^{\infty} \frac{5n+12}{n^2(n+2)}$ is convergent or divergent.
20. Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n+5)}{(n+4)(n+1)}$ absolutely convergent, conditionally convergent or divergent.
21. Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/5}}$ is convergent or divergent.
22. Find the Maclaurin series for $e^{3x} + \cos(2x)$ using any method.
23. Find the Maclaurin series for $e^{3x} + e^{2x}$ using the definition of a Taylor series.
24. Find the Taylor series for $\sin x$ by integrating the Taylor series for $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.
25. Evaluate $\int \cos(\pi x) \cos(4\pi x) dx$.
26. Evaluate $\int \frac{dt}{\sqrt{t^2 + 2t + 5}}$.
27. Evaluate $\int \frac{dx}{x + x\sqrt{x}}$.
28. Evaluate $\int \frac{3x^2 - 2}{x^2 - 2x - 8} dx$.
29. Using the Trapezoidal Rule with $n = 10$, approximate the integral $\int_1^2 e^{1/x} dx$.
30. A swimming pool is 20 ft wide and 40 ft long and its bottom is an inclined plane, the shallow end having a depth of 3 ft and the deep end, 9 ft. If the pool is full of water, find the hydrostatic force on one of the sides stretching from the shallow end to the deep end.
31. Find the general solution for $\frac{dy}{dt} = \frac{t}{t^2y + y}$.
32. Solve the initial value problem $\frac{dy}{dt} = \frac{1 - y^2}{y}$, $y(0) = -2$.
33. Find the general solution for $\frac{dy}{dt} - \frac{2}{t}y = t^3 e^t$.
34. Solve the IVP $\frac{dy}{dt} = -\frac{y}{t} + 2$, $y(1) = 3$.

35. A vat contains 500 gallons of beer at 6% alcohol by volume. Beer that contains 7% alcohol by volume is pumped into the vat at a rate of 3 gallons per minute. Also beer that contains 4% alcohol by volume is pumped into the vat at a rate of 5 gallons per minute. The solution is mixed and then pumped out at a rate of 8 gallons per minute. What is the percentage of alcohol after 1 hour?
36. A tank contains 100 L of fresh water. Brine that contains .01 kg of salt per liter is pumped into the tank at 7 L/min. The solution is mixed and then pumped out at the same rate. Find a) the amount of salt in the tank at time t and b) the amount of salt in the tank at 30 minutes.
37. Find dy/dx , and the equation of the tangent line at the point $t = 1$ for the parametric curve $x = e^{\sqrt{t}}$ and $y = t - \ln t^2$.
38. Find dy/dx , d^2y/dx^2 and determine for which values of t the curve is concave up. $x = 4 + t^2$, $y = t^2 + t^3$.
39. Find the length of the curve $x = e^t - t$ and $y = 4e^{t/2}$ for $-8 \leq t \leq 3$.
40. Find the surface area generated by rotating the curve from the previous problem about the y -axis on the interval $0 \leq t \leq 1$.
41. Given the point in the plane with rectangular (Cartesian) coordinates $(-2\sqrt{3}, 2)$, find four different sets of polar coordinates for this point so that the angle satisfies $-2\pi < \theta \leq 2\pi$.
42. Find the area inside the circle $r = 2 \cos \theta$ which is also below the line $\theta = -\pi/3$.
43. Find the area inside the cardioid $r = 2 + 2 \cos \theta$ and also inside the circle $r = 6 \cos \theta$.