

Final Exam Review

11.10 Taylor Series

Know the definition of a Taylor Series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$ and how to find Taylor series using the definition. We learned the Maclaurin series for e^x , $\cos x$, $\sin x$ and we already knew the Maclaurin series for $\frac{1}{1-x}$, $\arctan(x)$ and $\ln(1-x)$. We can manipulate these series to find other series using the techniques of substitution, differentiation and integration. Be careful - if a problem says to use the definition, you won't receive full credit if you use another technique, even if you have the right answer!

12.1 3D coordinates

Plot points, know the coordinate planes, find projections onto coordinate planes, find distance in 3 dimensions, know the equation for a sphere.

12.2 Vectors

Know the different ways to write vectors, how to find a position vector for a point and how to find the displacement vector between two points, add or subtract vectors and multiply vectors by a scalar. Know how to find the length/magnitude of a vector, and how to find a unit vector in the same direction. Use the components of a vector to find the force on an object.

12.3 The Dot Product

Know how to find the dot product of two vectors. An important property of the dot product is $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$. Also, $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$ where θ is the angle between the 2 vectors when they are tail to tail. Two vectors are orthogonal if and only if their dot product is 0. Know how to find the vector and scalar projections of one vector onto another.

12.4 The Cross Product

Know how to find the cross product of two vectors. $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$. Also $\mathbf{a} \times \mathbf{b}$ is orthogonal to both \mathbf{a} and \mathbf{b} . Be able to find a vector perpendicular to the plane containing 3 given points. Also know the formulas for area of a parallelogram and volume of a parallelepiped.

12.5 Lines and Planes

Know how to find the vector, parametric, and symmetric forms of the equation of a line. Remember, you need the position vector for a point on the line \vec{r}_0 and a direction vector \vec{v} for the line. Then $\vec{r} = \vec{r}_0 + t\vec{v}$. Know how to find the normal vector \vec{n} to a plane. The equation of a plane is $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$ where \vec{r}_0 is the position vector for a specific point on the plane and \vec{r} an arbitrary point on the plane. Know how to determine if two lines are parallel, intersecting or skew, and how to find the point where they intersect. Know how to determine if two planes are parallel or intersecting, and if so, what is the angle between them.

Problems

- Find a Taylor series for $\cos(-2x)$ by using the derivative formulas for the Taylor series.
 - Find a Taylor series for $\cos(-2x)$ by substituting into the appropriate Taylor series.
- Given that the Maclaurin series for $(5+x)^{-1}$ is $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{5^{n+1}}$ for $|x| < 5$, find the Maclaurin series for $(5+x)^{-2}$.
- Find a unit vector parallel to the vector $6\vec{i} - 4\vec{j} + 12\vec{k}$.
- Find the angle in radians between the vector $5\vec{i} + 4\vec{j} - 8\vec{k}$ and the vector $6\vec{i} - 3\vec{j} + 7\vec{k}$.
- Given the vectors $\vec{v} = 3\vec{i} - 2\vec{j} + 5\vec{k}$ and $\vec{w} = 4\vec{i} + 6\vec{j} + 7\vec{k}$, find $\text{Proj}_{\vec{w}}\vec{v}$, $\text{Proj}_{\vec{v}}\vec{w}$, $\text{Comp}_{\vec{w}}\vec{v}$, and $|\vec{v}| \cos \theta$ where θ is the angle between \vec{v} and \vec{w} .
- Find the equation of the plane determined by the three points $(3, 0, -2)$, $(11, -5, 2)$, $(3, 7, 4)$.
- Determine if the two lines are parallel, intersecting or skew. If they intersect, find the point of intersection.

$$\vec{r}_1(t) = (5t + 3)\vec{i} + (-2t + 7)\vec{j} + 4t\vec{k}$$

$$\vec{r}_2(s) = 8\vec{i} + (6s + 1)\vec{j} + (3s + 2)\vec{k}$$

- Determine if the planes are parallel or if they intersect, determine the angle between them.

$$3x - 2y + z = 1, \quad 2x + y - 3z = 3$$

- Find the area of the parallelogram determined by the vectors $\vec{a} = \langle 6, 3, -1 \rangle$ and $\vec{b} = \langle 0, 1, 2 \rangle$.
 - Find the volume of the parallelepiped determined by the vectors \vec{a} and \vec{b} from part a, as well as by $\vec{c} = \langle 4, -2, 5 \rangle$.
- Find the equation of the plane that contains the line $x = 3 + 2t$, $y = t$, $z = 8 - t$ and is parallel to the plane $2x + 4y + 8z = 17$.

Need more review problems? Check out the chapter reviews!! Chapter 11 Review: 45-51 and Chapter 12 Review: 1-13,15-20,23,24