

## Math 1272 Midterm 3 Review

**Hint:** When using a test or a theorem, make it clear you are checking the hypotheses and write out a solution that will likely include a sentence or two.

### 11.1 Sequences

“An ordered list”:  $\{a_n\}_{n=1}^{\infty} = a_1, a_2, a_3, \dots$ . A sequence  $\{a_n\}$  converges to  $L$  if we can make the terms  $a_n$  as close to  $L$  as we like by taking  $n$  sufficiently large. In other words, *eventually*  $a_n$  gets really really close to  $L$  for all values of  $n$  that are big. Otherwise, the sequence diverges to  $+\infty$  or  $-\infty$  or the limit DNE. A lot of times, taking the limit of a sequence is a lot like taking the limit of a function, but sometimes we need the Squeeze Theorem and the theorem about  $|a_n| \rightarrow 0$  as  $n \rightarrow \infty$ . Know how to find the limit of a geometric series  $\{r^n\}$ .

### 11.2 Series

“An infinite sum”, “adding up a sequence”:  $a_1 + a_2 + a_3 + a_4 + \dots = \sum_{n=1}^{\infty} a_n$ . Remember there are 2 sequences associated with a series: the sequence of terms  $\{a_n\}$  and the partial sum sequence  $S_n = \sum_{i=1}^n a_i$ . A series converges if the limit of the partial sum sequence exists and is finite. Otherwise, the series diverges. Know how to determine if a geometric series converges and find its sum. Know how to find the sum of a telescoping series or how to find the sum of a series given the partial sum sequence. This section also contains the **\*\*Test for Divergence\*\***

### 11.3 Integral Test

For a series  $\sum_{n=k}^{\infty} a_n$ , remember to check the hypotheses of the Test:  $f(x)$  (where  $a_n = f(n)$ ) must be continuous on  $[k, \infty)$  and must be eventually positive and decreasing on  $[j, \infty)$  where  $j > k$ . For decreasing, check either that  $f'(x) < 0$  or do a comparison: if  $x_1 < x_2$ , then is  $f(x_1) > f(x_2)$ ? Big result: The test for  $p$ -series.

### 11.4 Comparison Test

Remember to check the hypotheses: Show that either (i)  $a_n \leq b_n$  AND  $\sum b_n$  converges or (ii)  $a_n \geq b_n$  AND  $\sum b_n$  diverges to show (i)  $\sum a_n$  converges or (ii)  $\sum a_n$  diverges. You can tell if a series will converge or diverge by approximating its behavior using the dominate terms in the numerator and the denominator. Then all that's left is to do the formal comparison! If the comparison you want to make is in the “wrong direction” (i.e.  $b_n \geq a_n$  but  $\sum b_n$  diverges), use the limit comparison test!

### 11.5 Alternating Series

Alternating Series Convergence Test - always check that the two hypotheses are true. As with the integral test, there are two ways to see if the sequence  $\{b_n\}$  is decreasing: algebraically and examining  $f'(x)$  of the related function. If one hypothesis is not true, then the series *probably* diverges, so try the test for divergence.

## 11.6 Absolute Convergence, Ratio Test and Root Test

Be able to use the ratio and root tests to determine if a series converges or diverges. When the limit is 1, then the tests give no conclusion, and you will need to use a different test! Use the Ratio test for series that contain factorials and constants to a power of  $n$ . Use the Root Test when the terms of the series are raised to a power of  $n$ , but neither test works very well for series that are rational or algebraic functions of  $n$ .

## 11.8 Power Series

Power series are series with  $x$ 's in them, where  $x$  is a variable. Hence a power series is a function.

$\sum_{n=0}^{\infty} c_n(x-a)^n$  is a power series centered at  $a$ . They have a radius of convergence  $R$  such that the power series converges if  $|x-a| < R$ . Also power series have an interval of convergence, which means testing the endpoints of the interval given by  $-R+a < x < R+a$ .

## 11.9 Functions as Power Series

Using geometric series, we figured out that  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$  for  $|x| < 1$ . Now we can represent a function  $f(x)$  as power series by substitution into this formula, or by differentiating or integrating. For substitution, we put the function  $f(x)$  in the form  $\frac{1}{1-(u)} = \sum_{n=0}^{\infty} (u)^n$  which converges for  $|u| < 1$ . For differentiation and integration, evaluate term by term to find the new power series.

## Review Problems (Set 1)

1. Determine whether the sequence  $a_n = \frac{n^2+3n}{\sqrt{4n+9}}$  converges or diverges. If it converges, find its limit.
2. Find the sum of  $\sum_{n=1}^{\infty} \frac{(-2)^{n-2}}{3^n}$ .
3. Determine whether  $\sum_{n=1}^{\infty} \frac{n}{n^2-3}$  converges or diverges.
4. Determine whether  $\sum_{n=1}^{\infty} \frac{5\sqrt[3]{n^2}-3}{n^2}$  converges or diverges.
5. Determine whether  $\sum_{n=1}^{\infty} \frac{1}{n7^n}$  converges or diverges.
6. Determine whether  $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{5k+3}{9k+4}$  converges or diverges.
7. For what values of  $x$  does the power series  $\sum_{n=0}^{\infty} (-1)^n (n+1) 3^n x^n$  converge? For what values does it diverge? (Find the radius and interval of convergence.)
8. a. Find the powers series representation of  $f(x) = (5+x)^{-1}$ .  
b. Find the power series for  $(5+x)^{-2}$ .

**Set 2**

1. Determine if the sequence  $a_n = \frac{\arctan(n^2)}{6^{n+2}}$  converges or diverges. If it converges, find its limit.
2. Determine whether  $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^{3/2}}$  converges or diverges. If it converges, find the sum.
3. Determine whether  $\sum_{n=2}^{\infty} \frac{\sin n}{(2n+1)(2n-1)}$  converges or diverges.
5. Determine whether  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}3^n}{2n^2+4}$  converges or diverges.
6. Determine whether  $\sum_{k=1}^{\infty} \cos(\pi k) \frac{5k+3}{k^2+4}$  converges or diverges.
7. Find the radius and interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{10^n(x+2)^n}{(n+1)!4^{2n+1}}$ .
8. a. Find the power series representation of  $f(x) = \ln|2+x|$ .  
b. Find the power series representation of  $f(x) = \int \frac{x}{1+x^3} dx$

Need more review problems? Check out the Chapter 11 review! The *true-false quiz in the Chapter 11 review* is a great way to test your understanding of the concepts of sequences and series. Then problems 1-8,11-29,34,40-44,47,49 are all on the material covered on this exam.