

Strategy for Integration

1. Simplify. e.g. Distribute over some parentheses or use a trig identity.

$$\begin{aligned}\int u^6(u^4 + 2u^2 + 1) du &= \int u^{10} + 2u^8 + u^6 du \\ \int \frac{\cot \theta}{\csc^2 \theta} d\theta &= \int \frac{\cos \theta}{\sin \theta} \sin^2 \theta d\theta \\ &= \int \cos \theta \sin \theta d\theta\end{aligned}$$

2. Make a substitution. This may be an obvious substitution, when part of the function is $u = g(x)$ and another part is $du = g'(x)$, e.g.

$$\int 2r\sqrt{4+r^2} dr, \quad u = 4 + r^2, \quad du = 2rdr \quad \int \sqrt{u} du$$

The square root seems to suggest that a trig substitution might be necessary, but the substitution $u = 4 + r^2$ makes the problem a lot easier. Or the substitution may be less obvious:

$$\int (1 + \sqrt{x})^8 dx, \quad u^2 = x, \quad 2udu = dx \quad 2 \int u(1 + u)^8 du$$

Here the substitution "rationalizes" the expression.

3. Look at the form of the integral.

- Is it the product of powers of $\sin x$ and $\cos x$ or other trig functions? Try using an appropriate trig identity!
- Is it a rational function? Try partial fractions!
- Is it the product of a power of x and a trig, exponential or logarithmic function? Try integration by parts!
- Does the integrand have a radical of the form $\sqrt{\pm x^2 \pm a^2}$? Try using Trig Substitution!

4. If one technique doesn't seem to be working, try applying more than one.

$$\begin{aligned}\int \cos x \sqrt{1 + \sin^2 x} dx, \\ u = \sin x, \quad du = \cos x dx \\ \int \sqrt{1 + u^2} du \\ \text{Then } u = \tan \theta, \quad du = \sec^2 \theta d\theta \\ \int \sqrt{1 + \tan^2 \theta} \sec^2 \theta d\theta \\ \int \sec^3 \theta d\theta\end{aligned}$$

5. Try again! Don't get discouraged if your first try doesn't work!

6. Do a lot of problems! The more problems you figure out, the more experience you'll have and the more you'll gain intuition about how to solve integrals!

Practice Problems (taken from section 7.5):

$$5. \int_0^2 \frac{2t}{(t-3)^2} dt$$

$$6. \int \frac{x}{\sqrt{3-x^4}} dx$$

$$13. \int \sin^3 \theta \cos^5 \theta d\theta$$

$$27. \int \frac{dx}{1+e^x}$$

$$29. \int_0^5 \frac{3w-1}{w+2} dw$$

$$45. \int x^5 e^{-x^3} dx$$

$$63. \int \frac{\sin 2x}{1+\cos^4 x} dx$$