1 Introduction and Motivations

1.1 General Background on Flocking

Flocking, a universal phenomenon of multi-agent interactions, has gained increasing interest from various research communities in biology, ecology, robotics and control theory, sensor networks, as well as sociology and economics.

(i) (**Biology and Ecology**) The emergent behavior of bird flocks, fish schools, wolf packs, elephant herds, or bacteria aggregations, for example, has long been a major research topic in population and behavioral biology and ecology [3, 4, 5, 9, 14, 20, 21].

(ii) (**Robotics and Control**) The coordination and cooperation among multiple mobile agents (robots or sensors) have been playing central roles in sensor networking, with broad applications in military, environmental control, or various field tasks [11, 22].

(iii) (**Economy and Languages**) Emergent economic behavior, such as a common belief in a price system in a complex market environment, is also intrinsically connected to flocking. The emergence of a common language in primitive societies is yet another example of a coherent collective behavior emerging within a complex system [5, 6].

The present work can largely be categorized into the biology realm, and has been directly inspired by the recent mathematical works of Cucker and Smale [4, 5], as the title suggests. Mathematical abstraction and rigorous analysis are more focused herein than actual biological or physical realizability or feasibility. As in physics, the study of idealized models can often shed light on various observed patterns in the real world, *if* such models can indeed catch the very essence.

In biology and physics, the main goal of flocking study is to be able to interpret, model, analyze, predict and simulate various flocking or multi-agent aggregating behavior. Most existing works have been focusing on modeling and simulation [12, 23]. See, for example, the several important models investigated by Flierl et al. [9] (and their stochastic formulation). The more recent paper of Parrish et al. [15] also provides a comprehensive comparison among some major existing models and their governing variables (in the context of fish schooling). Quantitative analysis (as in [4, 5, 11]) on the asymptotic rates of emergence and convergence, on the other hand, has been relatively rare.

Mathematical efforts are gradually gaining strength in this multidisciplinary area. In the continuum limit, for example, there have been several recent efforts made by Bertozzi's group [20, 21], in which global swarming (i.e., with densely populated agents) patterns are modeled and analyzed via suitable spatiotemporal differential equations. Discrete-to-continuum limits of interacting particle systems have also been investigated by the same group [1, 8] recently. Consistent and generic mathematical analysis has been very much in an early stage for many biological aggregation phenomena. In the current paper, following the recent remarkable works of Cucker and Smale [4, 5] on flocking analysis, we attempt to make further extension along the same line.

1.2 Cucker-Smale Flocking Model

Given a flock of k agents (birds, fish, wolves, etc) labeled by $i = 1, 2, \ldots, k$, the Cucker-Smale flocking model is specified by the *nonlinear* autonomous dynamic system:

$$\begin{cases} \dot{x}_i(t) = v_i, \\ \dot{v}_i(t) = \sum_{j \in \mathcal{L}(i)} a_{ij}(x)(v_j - v_i), \qquad i = 1 : k, t > 0, \end{cases} \tag{1}$$

where $x_i(t)$ and $v_i(t)$ are 3D (3 dimensional, which is non-essential) position and velocity vectors at time t, $x = (x_1, \ldots, x_k) \in (\mathbb{R}^3)^k$, and $\mathcal{L}(i) \subseteq \{1, \cdots k\}$ denotes the subgroup of agents that directly influence