Halftoning via Perona-Malik Diffusion and Stochastic Flipping

Jackie Shen, Program in Applied, Computational and Industrial Mathematics, University of Minnesota

The Nature of Halftoning

Halftoning is the key process governing most binary or ternary printing devices. The major task involved is easy to describe: how to convey faithfully a multitude of shades or colors using only a few elementary ones, e.g., black ink dots?

In some sense, halftoning simulates quantum physics. The latter reveals that the *continuum* of the material world we *observe* actually emerges from the *discrete*, or *quantum*, building blocks of basic particles and their discrete states. Halftoning attempts to reverse such engineering of Mother Nature by designing models and algorithms to express *smoothly varying* tones via only a few *discrete* ones.

For maximal clarity, this letter focuses only on the halftone process of expressing continuous-tone (or contone) grey shades by simply turning *on* or *off* black dots, as in most BW inkjet printers. This letter describes a method to halftone an image by a new algorithm of error diffusion. For a given contone image u in [0,1], the method shows how to design its halftone version b, which is binary from {0,1} at each pixel. In case of printing, one may for convenience assume that b=0 deposits an ink dot, while b=1 leaves the spot blank.

Error Diffusion

Suppose at a pixel α, the contone value $u(\alpha)$=0.75, while the halftoned value $b(\alpha)$=0 or 1. The error $e(\alpha)=u(\alpha)-b(\alpha)$=0.75 or -0.25, which in either case is non-negligible. This typical scenario differs halftoning from any other design tasks in computer graphics or computer aided design (CAD), for which pointwise approximation errors often diminish when one employs high-order Fourier modes, polynomials, splines, or wavelets. In contrast, *pointwise* error evaluation appears *pointless* for halftoning. As for HVS, the errors must be *blurred* to escape the detectability of the naked eyes.

Such understanding has inspired one of the greatest halftoning methods invented by Floyd and Steinberg[1] called *error diffusion* (ED). At a current pixel α, the halftone error $e(\alpha)=u(\alpha)-b(\alpha)$ is distributed to its neighboring pixels β's in such a manner that *locally* the errors cancel out. Thus a typical ED algorithm often relies upon *four* entities: a path visiting all the pixels, a pixelwise decision rule for converting $u(\alpha)$ to $b(\alpha)$, local windows into which halftone errors are diffused, and their distribution weights.

Novel 2-Step Progressive Algorithm

The new halftoning algorithm[2] is iterative, with two steps relaying at each iteration. Suppose at step n, the current halftone image is b^n with error field e^n. First one diffuses the error field *ediff*=PM(e^n), where PM stands for the *Perona-Malik* diffusion process to be explained later. Next, the diffused error field *ediff* induces a *stochastic flipping* (SF) strategy that can further polish the halftone image b^{n+1}=SF(b^n | *ediff*). So the iteration goes till convergence.

The characteristics of the new algorithm are: (**a**) independence on the particular choices of visiting paths, local diffusion windows, or diffusion weights, (**b**) being progressive instead of aiming at single-pass completion, and (**c**) allowing straightforward parallel implementations.

Perona-Malik Diffusion (PM)

Diffusion is ubiquitous, e.g., heat diffusion and Brownian motions[3], and often *homogeneous* and *isotropic*, i.e., being the same everywhere and in every direction, as quantified by the celebrated heat equation: u_t=D Δu. For intelligent image enhancement, Perona and Malik[4] devised an image-adapted anisotropic diffusion mechanism in the form of

$$u_t = \nabla \cdot (D(|\nabla u|)\nabla u),$$

where the diffusivity D(·) depends upon the input image u so that diffusion across edges is discouraged, crucial for not messing up different objects in images.

The current algorithm employs a revised version of the PM diffusion for error diffusion: *ediff*=PM(e^n) at each step n, so that error flows are confined within each object, as the HVS does. Moreover, the PM diffusion is a parallel process in contrast to most sequential ones.

Stochastic Flipping (SF)

The diffused error field *ediff* contains valuable information on the performance of the current halftone version b^n. At any pixel α, a smaller *ediff*(α) signifies good performance of the current halftone, while a larger one flags deficiency. This key qualitative observation leads to the quantitative strategy of *stochastic flipping* b^{n+1}=SF(b^n | *ediff*) that constitutes the other important half of the algorithm.

The flowchart of this progressive 2-step halftone algorithm is depicted in figure 1.

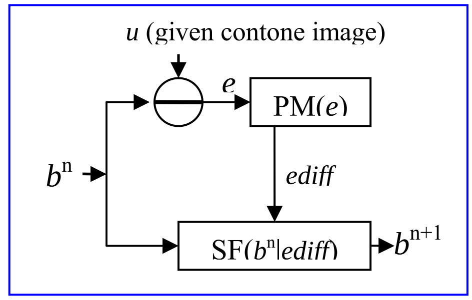

Figure 1. Flowchart of the progressive 2-step halftone algorithm.

Jianhong (Jackie) Shen
School of Mathematics, University of Minnesota, Minneapolis, MN 55455
Email: jhshen@math.umn.edu

Special Thanks to Dr. Gabriel Marcu at Apple Inc.

References:

1. R. Floyd and L. Steinberg, **Proc. Soc. Info. Disp., 17**, pp. 75-77, 1976.
2. J. Shen, **Proc. SPIE (on Electronic Imaging) 2006**, to appear.
3. T. F. Chan and J. Shen, "**Image Processing and Analysis - Variational, PDE, wavelets, and stochastic methods**," SIAM Publisher, Philadelphia, 2005.
4. P. Perona and J. Malik, **IEEE Trans. Pat. Anal. Mach. Intell.**, **12**, pp. 629-639, 1990.