# Math 4428 Problem Set 3

Assigned on: Friday, March 5; Due on Wed, March 24 (after the spring break)

| | |
|---|---|
| Chapters 13, 15 | Problems (© Jackie Shen, 2004) [ ***Prediction is the Power*** -Jackie ] |
| Advection: **Drug in Blood** | [**Drug Absorption in an Idealized Blood Vessel**] Consider an idealized 1-D blood vessel (from $x$=-œ to $x$=+œ) with constant flow speed $c$=2. A patient takes a certain type of drug to cure his disease. Suppose initially the concentration profile of the drug chemical is $f(x)= \exp(-x^4)$, and it is absorbed from the vessel to the ambient cellular environment at a rate of $r$=1. Assume that the system could be well modeled by the advection equation: $\boldsymbol{u_t = -c\, u_x - ru}$, with $\boldsymbol{u = u(x, t)}$ denoting the concentration at time t. Predict the drug concentration at $x$=1 and time $t$ =1. |
| Advection: **Wind and Rain** | [**Are You a Weatherman** ?] Idealize the US geography by thinking Minneapolis, Buffalo, and Boston as three spots in a 1-dimensional world (from $x$= -œ to $x$=+œ): $x_0$=0, $x_1$=2, and $x_2$=5. Today ($t$=0), the cloud concentration is measured to be $\boldsymbol{f(x)=1 + \cos(\pi x)}$. Suppose that (i). clouds are carried to the east (i.e. $x$>0 direction) by a steady wind of speed $\boldsymbol{c=7}$ (units); and (ii) due to heavy rain releasing, the cloud concentration decays at a rate of $\boldsymbol{r=2}$. Assume that this idealized weather system is well approximated by the advection equation in the previous problem. Predict tomorrow's ($t$=1) cloud concentrations at Boston. |
| Advection with **Variable Speed** | Consider the general advection model $\boldsymbol{u_t + q_x = -k}$. Suppose that $\boldsymbol{k=0}$ (i.e. lossless) and $\boldsymbol{q(x,t)=c\, u(x,t)}$, with $\boldsymbol{c=c(x)=2x}$ depending on the position. (1) Where will a particle initially at $x_0$ =4 be at time $t$=ln 2 (natural logarithm)? (2) What is the initial position of a particle being observed at $x$=1 at time $t$=ln 2? (3) Assume at time t=0, two particles sit at $x_0$ =1 and $x_0$ =2 separately with initial distance 1. Predict their distance at time $t$=ln 2. Explain intuitively why the distance expands. [Hint keywords: slower and faster.] |
| Advection: **Lossy or Lossless**? | Suppose $\boldsymbol{q(x,t)=c(x)\, u(x,t)}$ **and** $\boldsymbol{k(x,t)=r\, u(x,t)}$. If the overall lossy advection equation reads as: $\boldsymbol{u_t + 2x\, u_x = 0}$, what is the loss rate constant $r$? [Many people would think that there is no loss involved in this equation!] |
| Advection: **Total Population** | Consider a lossy advection system with an unknown $c(x)$ but known oscillatory loss rate $\boldsymbol{r=r(t)=\cos(t)}$, which is time dependent. Let<br>$\boldsymbol{Q(t)}$ = \integral $\boldsymbol{u(x, t)\, dx}$ from –infinity to +infinity<br>denote the total population at each time $\boldsymbol{t}$. Suppose initially $Q(0)$=1 (unit). What is the total population at time $\boldsymbol{t=\pi}$? |
| *Attention:* Midterm on Friday, March 12 will cover up to PS 3 (inclusive). Wed, March 10 will be **a review lecture** for the midterm. **Extra Office Hours**: 10:00am-12:00pm, Fri, March 12. | |