Math 4428: Problem 4, Monday, March 29. Due on: Wednesday, April 14, 2004.

<table>
<tr><td>Subjects</td><td>Problems (© Jackie Shen, 2004) [Life has randomness, and randomness has life]</td></tr>
<tr><td>Conditional Probability: Car accidents in Minneapolis</td><td>1. City government gives the following conditional probabilities of daily # of car accidents under different weather conditions during March in Minneapolis:
<table>
<tr><td>N=n</td><td>n=1</td><td>n=2</td><td>n=3</td><td>n=4</td></tr>
<tr><td>p(N=n | rain)</td><td>0.3</td><td>0.3</td><td>0.3</td><td>0.1</td></tr>
<tr><td>p(N=n | snow)</td><td>0.1</td><td>0.4</td><td>0.3</td><td>0.2</td></tr>
<tr><td>p(N=n | sunny)</td><td>0.6</td><td>0.3</td><td>0.1</td><td>0</td></tr>
</table>
The chances for each condition in a normal March are:
Prob(rain) = 0.3, Prob(snow) =0.3, and Prob(sunny) = 0.4.
(1.1) What is the chance p(2) for a given day in March to have n=2 accidents?
(1.2) On average in March, how many daily accidents could be expected?</td></tr>
<tr><td>Binomial Diffusion in Polarized Tissues</td><td>2. Let us model a piece of thin skin tissue by a 2-D lattice
{(n, m) : n, m =0, 1, 2, ...) }.
Suppose initially 1 million drug molecules are injected at (0, 0) for a skin test, and that each molecule moves independently by the following rules (within a unit time):
(a) with p=75% chance it moves to the right neighboring lattice point (i.e. n++);
(b) with q=25% chance it moves to the upper neighboring lattice point (i.e. m++);
Predict at time t=6, how many molecules could be observed at site (4, 2).</td></tr>
<tr><td>1-800-Service Station: Be a smart manager</td><td>3. You are just appointed as the manager for a 1-800 customer service call station which currently has 7 people answering incoming calls. Your research finds:
(a) #incoming calls can be well modeled by Poison P(100) (in one hour);
(b) on average, each incoming call takes 6 minutes to resolve its issue.
Should you consider hiring more people? How many openings in an optimal way ?</td></tr>
<tr><td>Exponential Neuron Firing</td><td>4. Suppose the time interval T for an idealized neuron to fire a new spike is well modeled by the exponential type E(10).
(4.1) What is the average firing rate per unit time?
(4.2) On average, how many firings could be expected within 10 units of time?
(4.3) What is the average waiting time for a new firing to occur?</td></tr>
<tr><td>Poisson Signaling</td><td>5. Two cells independently emit a same type of signaling chemicals (i.e. molecules) into the extra-cellular environment. Suppose they are well modeled by Poisson's P(10) and P(20) (within a unit time).
(5.1) On average, how long does it take for the combined system to emit 6000 signaling molecules?
(5.2) What is the probability type of the combined system: Gaussian, exponential, or still Poisson?
(5.3) What is the average waiting time for the combined system.</td></tr>
<tr><td>Sense and Sensibility: Selective firing</td><td>6. Suppose that the thumb's fingertip has n=10 temperature sensors, and when it touches a material surface (say wood, iron, etc):
(a) the number of sensors activated is subject to the binomial B(n=10, p=1/2);
(b) each sensor, when activated, starts to fire spikes independently according to Poisson P(5) (within a unit time).
All the spikes are then transmitted to the brain and counted by the central nerve system, allowing us to feel the relative temperature of the material. What is the probability distribution for the total number M of spikes (within a unit time) counted by the central nerve system?</td></tr>
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