Problem 5 [on Markov Chains] Assigned on: Friday, April 16, Due: Friday, April 30, 2004]

Subjects	Problems (© Jackie Shen, 2004) [***Out of Randomness Rise Structures and Predictability***]
Markov Chains: Trinity's Matrix: Neo Needs Help!	(**1**) State the Markov Transport Theorem, (**2**) from which derive the Markov Equilibrium Equation; Using her cellular phone, Trinity transfers to Neo the following 3 by 3 Markov transition matrix: **M=[0.3, 0.2, 0.1 (1st row); 0.2, 0.4, b (2nd row); 0.5, a, 0.5 (3rd row)]** . But the entries ***a*** and ***b*** are lost due to Agent Smith's interference. (**3**) Help Neo recover them. (**4**) Draw the three-state Markov transition graph associated to this matrix.
Markov Equilibrium of a Simple Organism	Consider a 4-cell organism with all cells linearly sitting at positions (0, 1, 2, 3) along x-axis. A certain type of nutrition particles randomly (and ***independently***) move through the cell membranes. Suppose that P(i+1 | i) = p, P(i-1 | i)=q, and P(i | i)=1-p-q, are the transfer probabilities for each individual particle, i=0:3, assuming that P(4|3)=0 (since no 4 exists) and P(-1 | 0)=0. Suppose under a laser beam, initially the cell at 0 produces 10 million particles. Afterwards the laser beam is removed and no particles are produced or destroyed. Predict the long-term distribution when (**case 1**): p=q=1/3; and (**case 2**) p =2q=1/2.
Capturing and Waiting Time: A Lazy Immune Cell vs. a Virus:	Consider a 1-D tiny cellular environment inside human body, which can be modeled by four sites along the ***x***-axis: 0, 1, 2, and 3. Suppose a lazy immune cell is immobile and always stays at site 0, while a virus is energetic and moves around. Whenever the virus moves to site 0, it will be captured and eaten up by the immune cell. Suppose the virus' random walk is described by : P(i-1 | i)= P(i+1 | i)=1/3, i=1, 2, 3, assuming that P(4|3)=0. Also notice that P(1 | 0)=0. Suppose the virus is initially at site 3. (**1**) What is the probability that the bacteria will be eaten up at some finite time T? (**2**) What is the average waiting time t_3=E[T_3] for the virus to be destroyed by the immune cell?
Lazy but Clever Immune Cells	Following the preceding problem, suppose in addition that our immune cell is smart and emits certain type of chemical signal, so that the transition probabilities of the virus is altered to: P(i-1 | i)=3/4 and P(i+1 | i)=1/4 (i.e. causing the virus to prefer the motion toward the immune cell). (**1, 2**) Answer the same two questions in the preceding problem. (**3**) Is the average waiting time t_3=E[T_3] indeed shortened?
Gene Regulated Network: Hi, Mr. Gene!!!!!! (Have you ever wondered how structures (limbs/head, etc.) could arise from a single cell?)	Ever wondered how Mr. Gene controls growth and creates body differences? Consider a simple linear cell divided into four sites, labeled by 0, 1, 2, and 3. Suppose its head is mainly made of protein **S**, while the distribution of **S** is controlled by Mr. Gene **G**. Suppose gene **G** has a ***fixed*** concentration distribution inside the cell: (G(0)=7, G(1)= 4, G(2)=2, G(3)=1) in certain units, meaning that, for example, 4 units of gene G are concentrated at site 1. Suppose initially protein **S** is distributed by (S(0)=2, S(1)=4, S(2)=4, S(3)=2), in millions, and that each protein molecule moves independently according to Markov transition law: P(i-1 | i) =(G(i-1)-G(i)) / (G(i-1)-G(i+1)); P(i+1 | i) = (G(i) - G(i+1)) / (G(i-1)-G(i+1)), i=1, 2, 3, assuming that G(4)=G(3) to use this formula for boundary site i=3, and P(1 | 0)=1/3, P(0|0)=2/3. Now you see how Mr. **G** has regulated the distribution of protein S ! What is the long-term eventual distribution of protein **S**? Is it uniform (i.e. same amount at each site)? Notice that biologically the head will develop where S is highly concentrated.
Note	Last problem set of Spring, 2004. Thank you all for making the course a fun and pleasure!