

Problem Set 1 (4428 Math Model) [Assigned on Monday, Feb 1; Due on Friday, Feb 13]:

Chapters/Themes	Pages	Problems (Last three problems © Jackie Shen 2004)
2	43	1(d), 2(b)[following 1(d) only]
3	62	1(b and c), 2 (b and c)
4	71	1
Extra	481	1
Exponential Law	Carbon dating	A piece of fish fossil was just unearthed somewhere in Africa. Measurement shows that its concentration of C^{14} is about 12% of that of the contemporary environment. Assume that the concentration in the environment has remained stable in history, and any life maintains the same concentration level of C^{14} as the environment before its death. Determine the age of this fish fossil. (The half-life time of carbon 14 is $t_{1/2}=5730$ years.) Clearly explain the model you use and its associated data.
Exponential Law with Constant Supply	Gandalf's checking account	UMN freshman Gandalf just opened a checking account at the US Bank with initial deposit \$5000. The generous bank manager gave him a fixed monthly interest rate of 0.1%. To pay for tips in restaurants (he often dines out), Gandalf plans to withdraw B dollars each month from the account on average. (1) Suppose $B=4$ dollars/month. What is the balance exactly four years later when Gandalf finishes his B.S.; (2) Determine the maximum value of B_{\max} , so that any more aggressive withdrawal B above it will eventually empty the account.
Logistic Growth Law	Aragorn's fish pond	Out of the 10000 lakes that Minnesota claims, our UMN junior Aragorn owns one of them with current fish population $p_0=8$ (units). After spending 4 years in measuring and modeling the lake, Aragorn's biology senior friend Gimli has concluded that the Logistic Model: $\frac{dp}{dt} = rp(m-p)$ works quite accurately for the fish population $p(t)$ without any human interference, with $r=0.01$, and $m=20$ (units). The time (i.e. t) unit is in years. (1) In such a natural environment, what are the precise fish populations one year later and 10 years later? (2) Suppose that from now on, Aragorn decides to sell C (units) of fish annually out of his lake. Help Aragorn to decide the maximum level C_{\max} , beyond which the fish population is put at the risk of eventual depletion.

Also available at course web: <http://www.math.umn.edu/~jhshen/4428-2004/>