

**Problem Set 2 [Assigned on Wed, Feb 18, Due on Mon, March 1, 2004]:**

Chapters/Themes	Pages/Topics	Problems (Population Dynamics of Two Species)
Generic Equilibria	Local Behavior	(a) Sketch the <b>local</b> dynamics for each generic equilibrium type: a stable node, an unstable node, a focusing (stable) spiral, a defocusing (unstable) spiral, and a saddle point. (b) For each generic equilibrium type, describe <b>qualitatively</b> the behavior of the associated two eigenvalues (i.e. real or complex, and their signs).
Eigenvalues	Real or Complex	Find the eigenvalues for each of the 2-by-2 matrices: $\mathbf{A} = [4, -1; -1, 4]$ $\mathbf{B} = [1, -2; 2, 1]$ . Here the semicolons denote row breaks.
Characteristic Polynomials	A Formula to Impress Jerry	(a) Express the characteristic polynomial of a 2-by-2 matrix $\mathbf{A}$ in terms of its trace $\text{tr}(\mathbf{A})$ and determinant $\det(\mathbf{A})$ . (b) Suppose via his cellular phone, Jerry transmits to you the following information $\text{tr}(\mathbf{A})= 7$ , $\det(\mathbf{A})=12$ of a 2-by-2 matrix $\mathbf{A}$ . But leaving in a rush for the movie <i>The Matrix</i> , he forgets to transmit to you the entire matrix $\mathbf{A}$ . Can you impress him by sending him the precise information of the two eigenvalues, even without knowing $\mathbf{A}$ ?
Linear Systems	Exact Solutions	Consider the linear system $\mathbf{p}'(t)=3\mathbf{p}-\mathbf{q}; \quad \mathbf{q}'(t)=-\mathbf{p} + 3\mathbf{q};$ <b>with initial conditions <math>\mathbf{p}_0=1, \mathbf{q}_0=2</math>.</b> Find the exact solution $\mathbf{p}(t)$ and $\mathbf{q}(t)$ . Is $(0, 0)$ a node, spiral, or a saddle point?
6/Null Clines	103/Nonlinear System	6
8/Null Clines	139/Linear System	5
5/Lotka-Volterra	81/Stability	Find ALL the equilibrium states and their stability types (i.e. stable or unstable) for the Lotka-Volterra equation (10) with $a=3, b=2, c=1, d=2, e=2$ .

**Attention:** The lecture on Wed, Feb 18 is crucially helpful for problems 1 through 4. Please borrow the lecture notes from your fellow classmates if you happen to miss it for some reason.