

Ask! Indicate your approach! Show your work! Good Luck! There are 8 problems, 5 pages, and 100 points.

(1) [15] Let  $V$  be the vector space of all continuous real-valued functions defined on the interval  $[0, 1]$  and let  $S$  be the set of all  $f \in V$  such that  $f(0) = 2f(1)$ . Express  $S$  in set notation and determine whether it is a subspace of  $V$ . What if  $S$  is defined by  $f(0)^2 = 4f(1)^2$ ?

(2) [10] Let  $\mathbf{p} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$  and  $\mathbf{q} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ . Find all pairs of scalars  $c, d$  such that  $\{\mathbf{p} + c\mathbf{q}, \mathbf{q} + d\mathbf{p}\}$  is LI.

- (3) [15] Find the dimension of the nullspace of  $A = \begin{pmatrix} 1 & 1 & 2 & 3 \\ 2 & 1 & 3 & 2 \\ 1 & 0 & 1 & -1 \\ 1 & -1 & 3 & 4 \end{pmatrix}$ .

- (4) [10] Does the set  $B = \left\{ \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \right\}$  span  $\mathbb{R}^3$ ? If not, find a subset of  $B$  that has the same span as  $B$ . If  $B$  does span  $\mathbb{R}^3$ , express  $\mathbf{e}_2$  as a linear combination of the vectors in  $B$ .

Scratch Page Be sure to **CLEARLY** link work here to a problem! Put the link **THERE** too!

(5) [10] Sketch what the matrix  $\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$  does to the rectangle determined by  $(1, 1)$ ,  $(3, 1)$ ,  $(3, 2)$ .

(6) [15] Find the eigenvalues and the eigenvectors for the matrix  $\begin{pmatrix} 2 & 4 \\ -3 & 10 \end{pmatrix}$ .

(7) [15] Find all scalars  $C$  such that the matrix  $\begin{pmatrix} 2 & 0 \\ C & 2 \end{pmatrix}$  is defective.

(8) [15] Diagonalize  $A = \begin{pmatrix} 3 & 2 & -1 \\ -2 & -1 & 2 \\ -2 & -2 & 4 \end{pmatrix}$ , given that  $A$  has eigenvalues 1, 2 and 3. In other words, find the  $S$  so that  $S^{-1}AS$  is a diagonal matrix.