

Ask! Indicate your approach! Show your work! Good Luck! There are 10 problems, 6 pages, and 120 points.

(1) [10] What is a matrix? Calculate all products (of pairs) possible among

$$(1 \ 2 \ 4), \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 1 \end{pmatrix}.$$

(2) [10] Write \mathbf{i} , \mathbf{j} and \mathbf{k} as column vectors. Express $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ in terms of \mathbf{i} , \mathbf{j} and \mathbf{k} .

(3) [10] In \mathbb{R}^3 , find a “scalar” equation, $ax + by + cz = d$, for the span of $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$. Hint: the origin is in the span. Be sure you find a valid equation!

(4) [10] Find a non-zero vector \mathbf{n} that is perpendicular to every vector in the plane of #(3).

(5) [10] Find the point in the plane in #(3) closest to $(1, 2, 3)$. Hint: Find t such that $\mathbf{p} := \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - t\mathbf{n}$ lies in the plane; \mathbf{n} is from #(4).

(6) [10] Given that A , B , C and D are points in \mathbb{R}^n , how can we check whether or not the directed line segments \overrightarrow{AB} and \overrightarrow{CD} are parallel?

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(7) [10] Find the dimension of the subspace V of \mathbb{R}^4 that is generated by the vectors

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

(8) [15] Suppose V and W are subspaces of \mathbb{R}^n . Why is $V \cap W$ always a subspace of \mathbb{R}^n , but $V \cup W$ is not always a subspace?

(9) [10] Suppose V and W are subspaces of \mathbb{R}^n . When is $V \cup W$ a subspace of \mathbb{R}^n ? Why?

(10) [10] Find all solutions of

$$\begin{aligned}2 &= x + 2y - z \\1 &= x - y + 2z \\-1 &= 2x + y - z.\end{aligned}$$