Ask! Indicate your approach! Show your work! Good Luck! There are 10 problems, 6 pages, and 120 points.

(1) [10] Suppose that $f : \mathbb{R} \to \mathbb{R}^n$ is differentiable and that $|f(t)|$ is constant. Show that $f'(t) \perp f(t)$.

(2) [10] Give an example of an open subset of $\mathbb{R}$ that has exactly one boundary point.

(3) [10] Find one polar-coordinate formula for two circles of radius one, having centers on the y-axis, that are tangent to each other along the x-axis. One circle lies below, the other above, the x-axis.

(4) [10] Find a basis for the kernel of the differential operator $D := \left(\frac{d}{dx}\right)^3 + \left(\frac{d}{dx}\right)^2 - 4\frac{d}{dx} - 4I$.

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(5) [10] Find the inverse of $\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 8 \\ 2 & 5 & 8 \end{pmatrix}$, if it has one. If not, find its rank.

(6) [10] A certain linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ maps $\mathbf{i}+\mathbf{j}$ to $\mathbf{j}+\mathbf{k}$, maps $\mathbf{j}+\mathbf{k}$ to $\mathbf{k}+\mathbf{i}$ and maps $\mathbf{i}+\mathbf{k}$ to $\mathbf{i}+\mathbf{j}$. Find the matrix for T.

(7) [10] Let V be the subspace of $\mathbb{R}^4$ that is spanned by the vectors

$$\begin{pmatrix}1\\2\\4\\2\end{pmatrix}, \quad \begin{pmatrix}2\\3\\8\\5\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}2\\5\\8\\3\end{pmatrix}. \text{ Find a basis for } V^{\perp}.$$

(8) [15] Find a basis for $\mathbb{R}^4$ that includes $\begin{pmatrix}2\\3\\5\\8\end{pmatrix}$ and $\begin{pmatrix}2\\5\\3\\8\end{pmatrix}$.

(9) [10] Find a basis for the image of $\begin{pmatrix} 1 & 2 & 4 & 2 \\ 2 & 3 & 8 & 5 \\ 2 & 5 & 8 & 3 \end{pmatrix}$.

(10) [10] Find a basis for the kernel of the matrix in (9).