Ask! Indicate your approach! Show your work! Good Luck! There are 10 problems, 6 pages, and 120 points.

(1) [10] Determine whether or not the function $f(x, y) = \begin{cases} \frac{(x+2y)^2}{x^2+y^2} & \text{if } x^2+y^2 \neq 0, \\ 0 & \text{if } x^2+y^2 = 0 \end{cases}$ is continuous at $(0, 0)$.

(2) [10] Given that $f(x, y) = \begin{pmatrix} x^2 - y^2 \\ 2xy \end{pmatrix}$ and that $g(u, v) = \sqrt{u^2 + v^2}$, find $(g \circ f)'$.

(3) [10] Find the derivative of the mapping $(x, y, z) \mapsto (r, \theta, \varphi)$, the conversion from rectangular to spherical coordinates.

(4) [10] Suppose that $f : \mathbb{R}^n \to \mathbb{R}^m$. Define f is *differentiable at* x_o, and explain why a function that is differentiable at x_o is continuous at x_o.

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(5) [10] Give an example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ that is continuous at every point (x, y) but that fails to be differentiable at some point (x_o, y_o). Justify your answer!

(6) [10] State the Chain Rule for vector-valued functions of a vector variable. Let $G(x, y)$ be defined for pairs of vectors in $\mathbb{R}^n$ by $G(x, y) = xy^T$. What is the derivative of the function $f(y) : \mathbb{R}^n \to \mathbb{R}^n$ defined by $f(y) = G(x, y)x$, where x is arbitrary but is fixed (you don't know what x is!)?

(7) [10] Determine whether or not the function $f(x, y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{if } x^2+y^2 \neq 0, \\ 0 & \text{if } x^2+y^2 = 0 \end{cases}$ is differentiable at $(0, 0)$. Justify your answer!

(8) [15] Find the first and second derivatives of $f(x, y, z) = xy + yz + zx$.

(9) [10] What rectangular parallelopiped of fixed perimeter has the greatest volume?

(10) [10] Find the extreme values, and where they occur, for the function $f(x) = x^3(x+7)^4$.