Ask! Indicate your approach! Show your work! Good Luck! There are 10 problems, 6 pages, and 100 points.

(1) [10] What is meant by the "Cauchy Product," $\sum_{n=0}^{\infty} c_n$, of two series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$? State conditions on the a and b series that guarantee that the Cauchy Product converges. What does it converge to?

(2) [10] Find the sum of $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. Hints: replace n^2 by x^n. Find the sum. Simplify. Differentiate. Multiply by x. Differentiate again. Set $x = 1$.

(3) [10] Find the radius of convergence r^* of the power series $\sum_1^\infty n^n z^{n^2}$. What is the convergence behavior of the series on the circumference of the circle of radius r^*?

(4) [10] Describe the convergence behavior of the series $\sum_0^\infty x^n$. Your answers will depend on x.

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(5) [10] State the Theorems on the integration and differentiation of series of functions.

(6) [10] Define $\bar{z}$ for complex numbers. Verify the formula $\overline{zw} = \bar{z}\bar{w}$.

(7) [10] Define *uniform convergence* for sequences of functions and for series of functions. Check your definitions on $[-1/2, 1/2]$ for $\{x^n\}$ and $\sum x^n$.

(8) [10] Find the first three non-zero Taylor coefficients of $\sec x$ when $x_o = 0$. Hint: Long Division works too!

(9) [10] State Taylor's Formula with Remainder. Apply it to $f(x) = \sin^2 x$. Does the Taylor *series* converge to $\sin^2 x$? Why?

(10) [10] Find the Cauchy Product of $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ and $\sum_{n=0}^{\infty} \frac{w^n}{n!}$. Does it converge? Why? If it converges, what does it converge to? What familiar formula does this give?