Ask! Indicate your approach! Show your work! Good Luck! There are 10 problems, 6 pages, and 130 points.

(1) [10] State and prove the Power Fact (about positive integer powers of positive numbers)

(2) [15] Prove that, if $\{z_n\}$ converges, then $\{z_n - z_{n+1}\}$ is a null sequence. Give an example of a sequence $\{z_n\}$ such that $z_n - z_{n+1} \to 0$, but such that $\{z_n\}$ does not have a limit. Your sequence does not have to be complex!

(3) [15] Prove that a complex sequence $\{z_n\}$ converges to z if and only if its sequences of real and imaginary parts converge to $\mathbf{Re}\ z$ and $\mathbf{Im}\ z$, respectively.

(4) [10] Suppose that the sequence defined recursively by $x_1 := 1$ and $x_{n+1} := \dfrac{1}{1+x_n}$ converges. What is the limit, assuming it exists?

(5) [10] Prove that, if $\{z_n\}$ and $\{z'_n\}$ are sequences with limits z and z' respectively, then for all complex numbers a and b, the sequence $\{az_n + bz'_n\}$ has limit $az + bz'$. Give an "epsilon-N" proof.

(6) [20] State the Theorem about the convergence of decreasing sequences.
Use the Theorem to prove: $0 < x < 1 \Rightarrow \lim_{n\to\infty} x^n = 0$.

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(7) [10] Suppose that A is a non-empty subset of $\mathbb{R}$, that $B \subseteq \mathbb{R}$ is bounded above, and suppose that $A \subseteq B$. Prove that $\sup A$ and $\sup B$ both exist, and that $\sup A \leq \sup B$.

(8) [10] Prove that the sequence $\{(-1)^n\}$ has no limit as $n \to \infty$.

(9) [10] Prove that $\lim_{n\to\infty} \frac{1}{2^n} = 0$.

(10) [20] Prove that, if S is a bounded non-empty set of real numbers and $\sup S \notin S$, then there exists a sequence $\{s_n\}$ such that $s_n \in S$ for all natural numbers n, and $\lim_{n\to\infty} s_n = \sup S$. Hint: Consider the numbers $\sup S - (1/n)$.