Ask! Indicate your approach! Show your work! Good Luck! There are 10 problems, 6 pages, and 120 points.

(1) [10] Prove that if $\{n_k\}$ is a strictly increasing sequence of natural numbers then $n_k \geq k$ for all natural numbers k (this shows that $n_k \to \infty$).

(2) [15] Prove that the Harmonic Series diverges. Do not use Cauchy Condensation.

(3) [15] State Cauchy's Condensation Test. Test $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$ for convergence.

(4) [10] State the Theorem relating absolute convergence and convergence. Give an example of a series that is convergent (why convergent?) but not absolutely convergent.

(5) [10] If $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ both converge to the same value, is it true that a_n and b_n converge to the same value? Justify your answer!

(6) [10] Test for convergence (identify any convergence tests that you use):

(a) $\displaystyle\sum_{r=0}^{\infty} \frac{1}{r!}$; (b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}, \quad x > 0$; (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^n}$.

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(7) [20] What is a Geometric Series? What are the partial sums of a Geometric Series? Which Geometric Series converge? Why?

(8) [10] State the limit versions of the Root Test and the Ratio Test. Test $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ $(x > 0)$ for convergence.

(9) [10] *Prove* that $\sum_{n=1}^{\infty}(b_n - b_{n+1})$ converges if $\{b_n\}$ is a convergent sequence, by calculating the partial sums. Find the sum of the series. You are proving that telescoping series converge.

(10) [10] Prove that, if $\sum_{n=0}^{\infty} a_n z^n$ and $\sum_{n=0}^{\infty} b_n z^n$ both converge for $|z| < R$, then for every pair c, d of complex numbers, $\sum_{n=0}^{\infty}(ca_n + db_n)z^n$ converges for $|z| < R$.