Ask! Indicate your approach! Show your work! Good Luck! There are 10 problems, 6 pages, and 130 points.

(1) [10] State the Well-Ordering Property of $\mathbb{N}$, and prove that the set of all natural numbers n such that $2^n > 1 + 2n + 3n^2$ has a smallest member.

(2) [15] State and prove the Difference-of-Powers Formula.

(3) [15] State the Archimedean Property of $\mathbb{N}$. Prove that the set of positive powers of 2 in not bounded above.

(4) [10] Prove that, if A and B are subsets of a set X, then:

$$A \cap B = A \cup B \text{ if and only if } A = B.$$

(5) [15] Let A denote the set of even positive integers, B the set of positive integers divisible by 3, and C the set of positive integers divisible by 5. Identify the sets $A \cap B$, $A \cap C$, $B \cap C$ in a similar way. Identify $(B \cap C) \setminus (A \cap B \cap C)$, $(A \cup B \cup C)^c$ in words as well. What is the smallest member of $(A \cup B \cup C)^c$?

(6) [15] Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function that is not a constant function. Let us define a relation $\mathcal{O}$ on $\mathbb{R}$ as follows. We'll say that "x obscures y" if $f(x) = f(y)$. That is, the ordered pair $(x, y) \in \mathcal{O}$ if and only if $f(x) = f(y)$. Prove that "obscures" is an equivalence relation.

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(7) [10] Show that $9^n - 1$ is a multiple of 8, for all $n \in \mathbb{N}$.

(8) [10] Prove that for all $n \in \mathbb{N}$, $1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$.

(9) [20] Suppose that $x > 0$ and $x^2 < 5$. Prove that there exists a positive real number $h < 1$ such that $(x + h)^2 < 5$. The reason for forcing $h<1$ is that then $h^2<h$.

(10) [10] State the Completeness Axiom. Define all the definable terms you use! Set theoretical terms we'll regard as undefinable.