

Ask! Indicate your approach! Show your work! Good Luck! There are 2 pages, and 50 points.

(1) [12] State the *Completeness Axiom* [6]. Let $S := \{x \in \mathbb{R} : 0 < x < 1\}$. Verify that $\sup S$ exists and show that $\sup S = 1$ [6].

If a set S of real numbers is non-empty and bounded above then there is a number $\sup S \in \mathbb{R}$ that is an upper bound for S and no number smaller than the number $\sup S$ is an upper bound for S .

The set $S := (0, 1)$ is non-empty: $1/2 \in S$. The number 1 is an upper bound for S because $x \in S \Rightarrow x < 1$. If $y < 1$ and $y \leq 0$ then $1/2 > y$ and $1/2 \in S$. If $y > 0$ then $y < \frac{y+1}{2} < 1$ and $\frac{y+1}{2} \in S$.

(2) [14] Use (and cite!) axioms to prove that for all real numbers a , $-(-a) = a$. Hint: $-a$ has an additive inverse!

There is a w so that $(-a) + w = 0$ (additive-inverse axiom) and additive inverses are unique (axiom or theorem, depending on version of axioms). But then

$$\begin{aligned} 0 &= (-a) + w \text{ Add. inv. axiom} \\ &= w + (-a) \text{ comm., so } w = -(-a). \end{aligned}$$

Also, a has an additive inverse, $-a$ and $-a$ is unique. That is,

$$0 = a + (-a) \text{ Add. inv. axiom, uniqueness}$$

Therefore a is a w that satisfies the equation $(-a) + w = 0$, so by uniqueness the two solutions, a and $-(-a)$, are equal.

(9) [24] Suppose that A and B stand for Mathematical Statements, and that $a := 1$ if A is True, and $a := 0$ if A is False, with similar definitions for B . Here are six logic formulas, and six numerical formulas involving the numbers a and b . For each logic operation, there is a formula that gives the correct truth value for the operation, 1 for True, 0 for false. Match the logical operation formulas with the correct numerical formulas. **Warning: Each incorrect match will cancel a correct one! However, your score won't be negative!** Please answer by filling in the blanks. For example, if you think that formula (4) is the correct formula for $A \wedge B$, the second logic formula, your answer will include (2)(4). Show your work!

$\sim A$	(1)(<u>6</u>)	$a + b - ab$
$A \wedge B$	(2)(<u>5</u>)	$1 + 2ab - a - b$
$A \vee B$	(3)(<u>1</u>)	$a + b - 2ab$
$A \Rightarrow B$	(4)(<u>4</u>)	$1 + ab - a$
$A \iff B$	(5)(<u>2</u>)	ab
$A \oplus B$	(6)(<u>3</u>)	$1 - a$

In the table below, “#1” refers to numeric formula 1, $a + b - ab$, and “3#” refers to logic formula 3, $A \wedge B$. Under #1 we have *TTTT*, which means “or,” which is logic formula 3#, so we put (3)(1) in the answers ...

a	b	#1	#2	#3	#4	#5	#6
1	1	1	1	0	1	1	0
1	0	1	0	1	0	0	0
0	1	1	0	1	1	0	1
0	0	0	1	0	1	0	1
—	—	3#	5#	6#	4#	2#	1#