Math 4606, Summer 2003 **Assignments**

Assignments are due at the start of class on the given Due date.

Please Note! *Special problems are like "term papers." They must be well-written, in ink, on standard 8.5 x 11 paper, and must be succinct - with exactly enough detail.*

Paper torn from spiral notebooks is not acceptable.
Err in the direction of slightly excessive detail at first,
but prolixity is not acceptable.

Assignment 7, Book Problems: Due to you.

A "virtual" assignment. . . 7.4.5 # 4, 7, 8; 8.1.4 # 2, 4, 12; 8.2.3 # 4, 5, 6, 7, 10;
7.1.3 # 5, 7, 9, 10; 7.2.4 1, 2, 7, 8, 11, 12, 13, 14; 7.3.4 # 2, 3, 4, 5, 7.

Special Problem 6: Due Aug 1

5.2.4 (pp164-165) # 5, 7.

Assignment 6, Book Problems: Due Aug 4

5.2.4 (pp164-165) # 6, 10, 11, 13; 5.3.4 (p176) # 7,8.

OPTIONAL: A Challenge Problem: Due August 5. *You must include the statement that you did not discuss this problem with anyone else!* However, you may use references to books or papers, citing them in detail. You may ask me questions about the meaning of parts of the Challenge Problem.

Definition: *A function $f(x)$ defined on an interval $I \subseteq \mathbb{R}$ is absolutely continuous on I if, for all $\epsilon > 0$ there exists $\delta > 0$ such that for all finite collections of non-overlapping intervals $[a_n, b_n] \subseteq I$, $1 \le n \le N$, if $\sum_{n=1}^{N}(b_n - a_n) < \delta$ then $\sum_{n=1}^{N}|f(b_n) - f(a_n)| < \epsilon$.*

Here, N is an arbitrary positive integer, and *all* positive integers N must be allowed; intervals I and J are *non-overlapping* if they have at most one point in common – $[0, 1]$ and $[1, 2]$ are non-overlapping, but $[0, 3]$ and $[2, 4]$ overlap.

Examples: on $\mathbb{R}$, $x^{1/3}$ is absolutely continuous but x^3 is *not* absolutely continuous.

(a) [5] If f is absolutely continuous on I, prove that f is uniformly continuous on I and that, if I is bounded, so is $f(x)$.

(b) [10] Prove that if f and g are absolutely continuous and bounded, then $f \cdot g$ is absolutely continuous. Find examples showing that the product of unbounded absolutely continuous functions may not be absolutely continuous.

(c) [20] Suppose that $f : [0, \infty) \to [0, \infty)$, f is continuous (from the right) at 0, $f(0) = 0$, $f(x)$ is increasing, and that $f(x)/x$ is decreasing on $(0, \infty)$. Prove that f is absolutely continuous on $[0, \infty)$ (you might wish to verify, for your own use, that $\sqrt{x}$ is a function of this type).

(d) [25] Prove that, if $f(x)$ is diffrerentiable on $(0, \infty)$, increasing, and $xf'(x)$ is bounded on $(0, \infty)$, then f is absolutely continuous on every compact interval $I \subseteq (0, \infty)$ (you might wish to verify, for your own use, that $\log x$ is a function of this type).

Special Problem 5: Due July 25

(a) Show that $\sqrt{x}$ is *not* differentiable from the right at $x = 0$.
(b) Show that $\sqrt{x}$ is uniformly continuous on $[0, \infty)$.

Assignment 5, Book Problems: Due July 28

4.2.4 (p138) # 11, 12; 5.1.3 # 5(first part), 9; 5.2.4 (pp 163-165) # 1, 2, 3.

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Special Problem 4: Due July 18

Suppose that $\{x_n\}$ is a sequence of points in a (non-empty!) set S and $x_n \to c \in \mathbb{R}$. Prove that $c \in S$ or c is both a limit point and a boundary point of S. Use this to show that the limit of a convergent sequence of points lying in a closed set is in the set.

Assignment 4, Book Problems: Due July 21

4.2.4 (p138) # 3, 4, 6, 10. 5.1.3 # 1, 2.

Assignment 3, Book Problems: Due July 14

4.1.5 (pp125–127) # 1, 7, 14, 15; 4.2.4 (p138) # 1, 2.

Comments: In # 15, it should say *bounded* open interval. The only finite open interval is the empty set [$\emptyset = (1, 1)$]. *Interior (point)* of a set is defined on page 96; the interior of a set is the set of all that set's interior points. *Neighborhood* is defined and discussed nicely on pages 90 and 91.

Special Problem 3: Due July 8

Definition *A point y is a boundary point "for" a set $S \subseteq \mathbb{R}$ if, for every $\delta > 0$, $(y - \delta, y + \delta)$ contains at least one point in S and contains at least one point in S^c.*

A point that is a boundary point "for" S is also a boundary point "for" S^c. Usually we say "y is a boundary point *of* S." The boundary points of $[0, 1]$ are 0 and 1. The boundary points of $\mathbb{N}$ are exactly the points of $\mathbb{N}$.

Prove that a set $S \subseteq \mathbb{R}$ is closed if and only if every boundary point of S belongs to S.

Assignment 2, Book Problems: Due July 7

3.1.3 (p84) # 9; 3.2.3 (p98) # 1, 4, 5, 13; 3.3.1 (p106) # 1.

Assignment 1, Book Problems: Due June 27

3.1.3 (p84) # 1, 3, 4, 6, 7.

Special Problem 2: Due June 30

3.1.3 # 2. Prove your answers.

Assignment 0, Book Problems: Due June 17

1.1.3 (try them, and at least have comments ready)
1.2.3 (read and prepare some questions to ask about some or all of them)

Special Problem 1: Due June 19

Express all 16 possible "A–B" Truth Tables as formulas using "symbols" from the list

$$\{\ A,\ B,\ \text{not, and, or, implies, if and only if}\ \}.$$

Explain your answers clearly. Scoring will be competitive! Neatness, legibility, cogency all count!