Math 4606, Summer 2004 **Assignments** are due at the start of class on the given day

Note: Special Problems are like Term Papers. They must be well and neatly written, or typed, on standard size paper. Paper torn from spiral notebooks will not be accepted! Margins of one inch at least on all four edges and legibility are required. Special Problems will to some extent be scored competitively. The use of lined paper and writing on both sides of a page are quite acceptable.

You must acknowledge the help you receive from others! If an answer looks a lot like like someone else's, and the connection is not mentioned, I'll give zero points to the ones that follow the first one read! The same goes for answers that appear to be copied from a book: the book gets the points! If this happens to you, see me about it at once.

Special Problem 9: Due August 4

Section 8.3, # 4, 8, 14. Justify your answers!

Special Problem 8: Due August 2

Suppose that $f : [a, b] \to \mathbb{R}^N$, where $N > 1$ and that f is continuous on $[a, b]$ and differentiable on (a, b). Find an example of such an f that shows that the Mean Value Theorem does not hold when $N > 1$. However, prove that for this type of f it is true that $\|f(x) - f(y)\| \leq \sup\{\|f'(t)\| : t \in (\min\{x, y\}, \max\{x, y\})\} \cdot |x - y|$. This is a substitute for the *MVT*. You might find it useful to work with $g(t) := f(t) \bullet \Delta$, where Δ is a cleverly chosen vector relevant to the problem.

Assignment 6, Book Problems: Due July 29

Section 5.3, # 4; Section 6.1, # 9; Section 6.2, # 10.

Special Problem 7: Due July 26

Suppose that $f(x)$ is defined and differentiable on $(x_o - \delta, x_o + \delta)$ for some positive δ and that f' is differentiable at x_o. Prove that there exists a function $\zeta(h)$ defined for $|h| < \delta$ that is continuous at $h = 0$, with $\zeta(0) = 0$, such that for all $h \in (-\delta, \delta)$ $f(x_o + h) = f(x_o) + f'(x_o)h + \frac{f''(x_o)}{2}h^2 + \zeta(h)h^2$. Generalized MVT, variable h.

Assignment 5, Book Problems: Due July 21

Section 4.1, # 6, 18; Section 4.2, # 6, 14.

Special Problem 6: Due July 19

This one is worth 15 points! Prove the *Sequences are good enough for continuity* Theorem: *Given that* $f : D \to \mathbb{R}$ *and* $x_o \in D$, f *is continuous at* x_o *if and only if for every sequence* $\{x_n\}$ *such that* $x_n \in D$ *for all* n *and such that* $x_n \to x_o$, $f(x_n) \to f(x_o)$ [9]. *State a corresponding result – Sequences are good enough for limits – for "f has a limit as* $x \in D \to x_o$*"* [6]. You should cite a Special Problem to prove half of the first result! Caution: the "corresponding result" is not a trivial modification of the first one! No proof is required, but you should try to prove your statement "in your head" to be sure it's right. You might want to use $f(x) := (x^3 - 1)/(x^2 - 1)$, with $x_o = 1$, as an example.

Assignment 4, Book Problems: Due July 14

Section 3.5, # 5, 7; Section 3.7, # 12, 14.

Special Problem 5: Due July 13

This one is worth 15 points!
Make axiomatic proofs for the following statements:
(a) In every ordered field F, $x > 0 \iff -x < 0$ [3].
(b) In every field F the statement $(\forall a \in F)(\forall b \in F)(\forall c \in F)([a \neq 0 \wedge ab = ac] \Rightarrow b = c)$ is true [3]
(c) In every field F, if $xy = 0$ then at least one of x and y must be zero [4].
(d) In every ordered field F, if $x > 0$ then $x^{-1} > 0$. [5].

Special Problem 4: Due July 8

Given: there is a sequence $\{r_n\}$ such that every rational number in $(0, 1)$ appears in $\{r_n\}$ exactly once, and every r_n is a rational number in $(0, 1)$.

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Given: For all real numbers a and b, if $a < b$ then there exists a rational number r such that $a < r < b$.

Prove that for every $\bar{x} \in (0, 1)$, whether rational or irrational, there exists a subsequence of $\{r_n\}$ that converges to $\bar{x}$. Hint: Use the "Axiom of Continuity" Theorem.

Assignment 3, Book Problems: Due July 7

Section 2.4, # 4, 8; Section 2.5, # 3, 6; Section 3.1, # 2, 3.

Assignment 2, Book Problems: Due June 30

Section 2.1, # 8, 15; Section 2.2, # 1, 3; Section 2.3, # 3 (with $f(0) = 0$), 9, 14.

Special Problem 3: Due June 29

Prove Theorem 2.13 on page 42.

Special Problem 2: Due June 24

Prove Theorem 2.6 **directly,** that is, without using any other Theorems. Just use epsilons and deltas and algebra and definitions. I suggest you use the *methods* used in Monday's class to prove the limit theorem for $1/x$, at $x_o \neq 0$.

Assignment 1, Book Problems: Due June 23

Section 1.2, # 3, 6, 7, 8, 9, 10.

Special Problem 1: Due June 18

Write out the Truth Tables for the statements "Exactly one of A, B and C is true" and "At most one of A, B and C is true."