Math 5583, Fall 2004 **Assignments** are due at the start of class on the given day

Note: Special Problems are like Term Papers. They must be well and neatly written, or typed, on standard size paper. Paper torn from spiral notebooks will not be accepted! Margins of one inch at least on all four edges and legibility are required. Special Problems will to some extent be scored competitively. The use of lined paper and writing on both sides of a page are quite acceptable.

Special Problem 0: Due date: open so far

The *partial sums* $s_n(z) := \sum_{k=0}^{n} \frac{z^k}{k!}$ of $\exp(z) := \sum_{k=0}^{\infty} \frac{z^k}{k!}$ each have exactly n complex roots (counting multiplicities). Since $exp(z)$ (as we will show later) has no zeroes, the roots of the partial sums should tend to infinity. This has been discussed in the literature (see *The Zeros of the Partial Sums of the Exponential Series* by Peter Walker, American Mathematical Monthly, April 2003, pp 337–339). The zeroes in fact grow linearly, so writers actually look at the sets of roots of $p_n(z) := s_n(nz)$. They have found that the zeroes of the p_n "cluster" more and more closely near the set $K := \{z : |ze^{1-z}| = 1, |z| \leq 1\}$. Discuss this mathematically in some detail (proofs, examples (based on literature is OK)) or find approximate roots and illustrate them with the aid of computer graphics and mathematics programs.

Project, Due Dec 10

Devise a Special Problem that reflects your interests or your field. I can help you choose.

Assignment 13, Book Problems: Due Dec 8

§74, # 11, p 266; §77, # 1, p 276; §78, # 5, p 280; §80, # 6, p 286.

Special Problem 5: Due Dec 6

Find all the singularities in $\mathbb{C}$ of $\cot \pi z^2$ and the residue at each singularity.

Assignment 12, Book Problems: Due Dec 1

§64, # 1, p 230. Feel free to use the Theorem in §66.

Assignment 11, Book Problems: Due Nov 24

§60, # 7, #8, #11, p 214-215; §50, # 2, #10, p 172-173.

Assignment 10, Book Problems: Due Nov 17

§54, # 8, p 190; §56, # 1 (with $\cos\left(\frac{1}{z^2}\right)$) and #4 (with $f(z) = \frac{1}{z^3(1-z)}$), p 198.

Assignment 9, Book Problems: Due Nov 10

§46, # 2bc, 4, p 153-154; §48, # 6, p 163.

Special Problem 4: Due Nov 8

Verify that if f is analytic in a disc that contains a circle C then for every z that lies *inside* C and for every $n \in \mathbb{N}$, $f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(\zeta)}{(\zeta - z)^{n+1}} d\zeta$. Hints: For the induction step replace f by f' and integrate by parts.

Assignment 8, Book Problems: Due Nov 3

Here are some contour integrals for you to evaluate: (1) §40, # 7, p 129; (2) $\int_C \frac{dz}{z^2 - z_o^2}$, where $z_o \neq 0$ and C is the circle with center 0 and radius $2|z_o|$; (3) $\int_C \frac{dz}{z^2 - z_o^2}$, where $z_o \neq 0$ and C is the circle with center z_o and radius $|z_o|$; (4) $\int_C \frac{z\,dz}{1 + z^2}$, where C is the circle with center $1 + i$ and radius $\sqrt{2}$; (5) $\int_C \frac{dz}{1 - \sqrt{z}}$, using the branch of $\sqrt{z}$ with $\sqrt{1} = 1$, where C is the circle with center 1 and radius $1/2$; (6) $\int_C \frac{dz}{1 - \sqrt{z}}$, using the branch of $\sqrt{z}$ with $\sqrt{1} = -1$, where C is the circle with center 1 and radius $1/2$.

Hints: use Partial Fractions, algebraic tricks and the Cauchy Integral formula if possible; in (4) note that $1{=}{-i^2}$.

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Assignment 7, Book Problems: Due Oct 27

§28, # 6, p 89; §30, # 8, p 94; §33, # 15, p 104; §40, # 10, p 130.

Special Problem 3: Due Oct 22

If $\{c_n\}$ is a sequence of complex numbers we define R, $0 \le R \le +\infty$, by

$$\frac{1}{R} := \limsup_{n\to\infty} |c_n|^{1/n} \text{ and we call } R \text{ the } \textit{radius of convergence} \text{ of the power series } \sum_{n=0}^{\infty} c_n z^n.$$

Verify that if $|z| < R$ the power series converges absolutely and that if $|z| > R$ the power series diverges.

Assignment 6, Book Problems: Due Oct 20

§28, # 2, p 89; §30, # 3, p 94; §31, # 4, p 96.

Assignment 5, Book Problems: Due Oct 13

§28, # 1, p 89; §30, # 1, p 94; §31, # 1, p 96; Find a series $\sum_{n=0}^{\infty} c_n z^n$ that is equal to $\frac{1}{1+z^2}$ for $|z| < 1$ (i.e., find the c_n). Hint: Use $z^2 = -(-z^2)$ instead of Taylor's Formula!

Assignment 4, Book Problems: Due Oct 6

§22, # 10, p 69–70; §24, # 1, 2, 6, p 73–74; §25, # 7, p 79.

Special Problem 2: Due Oct 4

Using the polar formula for $\sqrt{z}$, $z \neq 0$, check analyticity. Suggestion: read about the polar version of the Cauchy-Riemann equations.

Special Problem 1: Due Sept 20

Given $z = x + iy \neq 0$ use computation (algebra, the quadratic formua, etc.) to find all $w = u + iv$ such that $w^2 = z$. In this problem, an ambiguous sign $(\pm)$ that appears in one part of your answer must agree with all other ambiguous signs. We use $\mp$ to denote the sign opposite to $\pm$. Do not use the polar form!

Assignment 3, Book Problems: Due Sept 29

§11, # 3 (with $x^2 + y^2 - 2y + i(2x - 2y)$) , p 35; §13, # 2, 5, p 42; §19, # 3, 9, p 60.

Assignment 2, Book Problems: Due Sept 15+x also!

§7, # 6d, 7, 9, 10, 11, p 21-22; §9, # 7, 8a, p 29; §10, # 4, 5, 7, p 31-32.

Assignment 1, Book Problems: Due Sept 15+x

§3, # 8, p 8; §4, # 3, 4, 5, p 11; §5, # 2, 15, p 13.