Ask! Indicate your approach! Show your work! Good Luck!

Define "function of bounded variation on $[a, b]$, " and show that the Dirichlet function is NOT a function of bounded variation on $[0, 1]$.

Define "f is Riemann-Stieltjes integrable with respect to a function α on $[a,\ b]$." Define your terms!

Define "uniform convergernce" and show that, if a sequence $\{f_n(x)\}$ of function s converges uniformly to $f(x)$ on $[a, b]$, and if each f_n is continuous a.e. and bounded, then so is $f(x)$. You should use other theorems to prove this; allude accurately to whatever you use.

Outline the proof that, if f is bounded and continuous a.e. on $[a, b]$, then f is Riemann integrable on $[a, b]$.

Show directly that if $\alpha(x) := [0 < x < 1]$, then $\int_0^2 2\alpha(x)\, d\alpha(x)$ does *not* exist, while $\int_0^2 d\alpha(x)^2$ *does* exist.

State Taylor's Theorem. Give Taylor's Formula using 3 terms (including the remainder) when $f(x) = \sqrt{x}$ and the "base point" is $\alpha = 1$. In the remainder term, find the point at which the second derivative is evaluated.

Show that, if $f(x) \le g(x)$ a. e., and both f and g are Riemann integrable on $[a, b]$, then $\int_a^b f(x)\, dx \le \int_a^b f(x)\, dx$. What form does the triangle inequality take for Riemann integrals?

State the "integration-by-parts" formula for Riemann-Stieltjes integrals and give either a proof or a complete outline of a proof.

Use Riemann sums to calculate $\int_0^x t^n\, dt$, when n is a positive integer.

Show that the characteristic function (indicator function) of a subset E of $[a,\ b]$ is Riemann integrable if the boundary of E is a set of measure zero. Is this necessary?

Give an example of sequences $\{f_n\}$ and $\{g_n\}$ that converge uniformly on a set E such that $\{f_n g_n\}$ does not converge uniformly on E.

If a sequence of functions continuous on $[0,\ 1]$ converges uniformly on $(0,\ 1)$, must it converges uniformly on $[0,\ 1]$? Why?

Discuss the convergence behavior of the sequence $\{f_n\}$ on $(0,\ \infty)$, where $f_n(x) = x/(1 + nx)$.

Outline the proof that $e^{z+w} = e^z e^w$, where e^z is given by its power series.

State the **Weierstrass** Theorem, and **outline** a proof.

Outline the proof that there exists a nowhere differentiable function on $\mathbb{R}$.

Give an example of a pointwise bounded sequence of continuous functions on $[0,\ 1]$ that has no uniformly convergent subsequence.

Prove that the derivative of $\sin x$ is $\cos x$.

Show that $\displaystyle\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} |a_{nm}| = \sup_{F \text{ finite}} \sum_{(n,m)\in F} |a_{nm}|.$

Anent the Final Exam:

Study the homework problems, Special problems, practice problems, Tests and online notes (especially where it says "Why?" or "you should" or "left to you," and so on).

Here are some problems since Test 2 to work on.

Show that the Gamma function is continuous, using Exercise 12 in Chapter 7.

A square matrix A is invertible if there exists a matrix B such that $AB = I = BA$, where I is the $n \times n$ identity matrix (that has ones on the diagonal and zeroes elsewhere). Show that in the finite case, each of the two equations implies the other, but that there exist $\mathbb{N}$ by $\mathbb{N}$ matrices A and B such that $AB = I$ but $BA \ne I$.

Show that, if A is $n \times n$ and invertible then its rows (transposed) form a basis for $\mathbb{R}^n$. Show the same for the columns. What is the change-of-basis matrix (between the columns and the transposed rows)?

Give an example of a double series for which changing the order of summation changes the sum.

Suppose we know that changing the order of summation changes the sum of a certain double series. Suppose that at least one of the sums is finite. Prove that the series can be rearranged to give any desired sum. You may take it for granted that the same statement is valid for single series.

Prove that, if $f : \mathbb{R}^n \to \mathbb{R}^m$, and $f(x_o + h) = f(x_o) + Mh + o(h)$, then $M_{ij} = \dfrac{\partial f_i}{\partial x_j}$, evaluated at x_o.

The Chain Rule can be extended to the multi-variable case. State this extension, and prove it.

Let $r := |x|$. Show that, if we define $\Delta := \sum_{i=1}^n \dfrac{\partial^2}{\partial x_i^2}$ (a *partial differential operator* called the *Laplacian),* then for each $n > 2$ there exists $\alpha > 0$ such that for all $r > 0$, $\Delta r^{-\alpha} = 0$.

Is $f(x, y) := \begin{cases} \dfrac{xy^2}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0); \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$ continuous? Bounded? Differentiable? Where? Why?

Is $f(x, y) := \begin{cases} \dfrac{x^2y^3}{x^2 + y^6} & \text{if } (x, y) \neq (0, 0); \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$ continuous? Bounded? Differentiable? Where? Why?

Find the matrix-norm (with respect to the Euclidean metrics) of a two-by-two matrix of real numbers.

Give an example of a function $f(x, y)$ that tends to zero along lines passing thru the origin, is continuous everywhere else besides the origin, and has no limit at the origin.