Ask! Indicate your approach! Show your work! Good Luck! There are 5 pages, and 100 points.

(1) [10] Define "function of bounded variation on $[a, b]$, " and show that the Dirichlet function is NOT a function of bounded variation on $[0, 1]$.

(2) [10] Define "f is Riemann-Stieltjes integrable with respect to a function α on $[a, b]$." Define your terms!

(3) [10] Define "uniform convergernce" and show that, if a sequence $\{f_n(x)\}$ of function s converges uniformly to $f(x)$ on $[a, b]$, and if each f_n is continuous a.e. and bounded, then so is $f(x)$. You should use other theorems to prove this; allude accurately to whatever you use.

(4) [10] **Outline** the proof that, if f is bounded and continuous a.e. on $[a, b]$, then f is Riemann integrable on $[a, b]$.

(5) [10] Show directly that if $\alpha(x) := [0 < x < 1]$, then $\int_0^2 2\alpha(x)\,d\alpha(x)$ does *not* exist, while $\int_0^2 d\alpha(x)^2$ *does* exist.

(6) [10] State Taylor's Theorem. Give Taylor's Formula using 3 terms (including the remainder) when $f(x) = \sqrt{x}$ and the "base point" is $\alpha = 1$. In the remainder term, find the point at which the second derivative is evaluated.

(7) [10] Show that, if $f(x) \leq g(x)$ a. e., and both f and g are Riemann integrable on $[a, b]$, then $\int_a^b f(x)\,dx \leq \int_a^b f(x)\,dx$. What form does the triangle inequality take for Riemann integrals?

(8) [10] State the "integration-by-parts" formula for Riemann-Stieltjes integrals and give either a proof or a complete outline of a proof.

(9) [10] Use Riemann sums to calculate $\int_0^x t^n\,dt$, when n is a positive integer.

(1) [0] 10 Show that the characteristic function (indicator function) of a subset E of $[a, b]$ is Riemann integrable if the boundary of E is a set of measure zero. Is this necessary?