Ask! Indicate your approach! Show your work! Good Luck! There are 5 pages, and 100 points.

(1) [10] Give an example of sequences $\{f_n\}$ and $\{g_n\}$ that converge uniformly on a set E such that $\{f_n g_n\}$ does not converge uniformly on E.

(2) [10] If a sequence of functions continuous on $[0, 1]$ converges uniformly on $(0, 1)$, must it converges uniformly on $[0, 1]$? Why?

(3) [10] Discuss the convergence behavior of the sequence $\{f_n\}$ on $(0, \infty)$, where $f_n(x) = x/(1+nx)$.

(4) [10] **Outline** the proof that $e^{z+w} = e^z e^w$, where e^z is given by its power series.

(5) [10] State the Stone-Weierstrass Theorem(s).

(6) [10] Suppose that $\sum_{n=0}^{\infty} c_n z^n$ converges for $|z| < R$ to a function $f(z)$. Prove that $f'(z)$ exists for $|z| < R$ and that $f'(z) = \sum_{n=1}^{\infty} n\, c_n z^{n-1}$ for $|z| < R$.

(7) [10] **Outline** the proof that there exists a nowhere differentiable function on $\mathbb{R}$.

(8) [10] Give an example of a pointwise bounded sequence of continuous functions on $[0, 1]$ that has no uniformly convergent subsequence.

(9) [10] *Prove* that the derivative of $\sin x$ is $\cos x$.

(10) [10] Show that $\displaystyle\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}|a_{nm}| = \sup_{F \text{ finite}} \sum_{(n,m)\in F} |a_{nm}|$.