Math 8601, Fall 2003 Assignments

Assignments are due at the start of class on the due date. Further Problems may be scored competitively! Neatness, legibility and cogency count! Give credit for help received, including books and hints from me and others; mention discussions. If a Further Problem is difficult, please include a "narrative" that tells what you went thru to reach your results. Further Problems should be written on standard size paper, and spiral-bound paper must be trimmed! There should be one-inch margins all around (name and problem number in the top margin are OK). Lined paper is fine, and using both sides is fine too. But if your handwriting fills up lines, double-space!

Further Problem 7: Due Dec 12: no late papers acepted!

If f is locally integrable on $\mathbb{R}^n$ prove that for any fixed cube Q with center zero, $\frac{1}{|Q|}\int_{x+Q} f$ is continuous as a function of x. Use this to prove, or prove directly, the claim made in the sentence following **(7.6)**. A function is *locally integrable* if each point $x \in \mathbb{R}^n$ has a neighborhood on which f is integrable (so it follows that f^* is lower semicontinuous, hence measurable).

Further Problem 6: Due Dec 5

Find explicit formulas for the Hardy-Littlewood maximal function of the characteristic function of an interval I centered at 0 $(|I| > 0, \quad n = 1,\ 2)$.

Assignment 13 & last
Book Problems: Due Dec 8

Chapter 7, # 3(second part), 4, 6, 7, 17.

Assignment 12
Book Problems: Due Dec 1

Chapter 5, # 11, 21; Chapter 7, # 3(first part).

Assignment 11
Book Problems: Due Nov 24

Chapter 5, # 4, 6, 13, 15 ($\omega(\alpha)$ finite and decreasing), 20.

Assignment 10
Book Problems: Due Nov 17

Chapter 4, # 19, 20 (f finite a.e.); Chapter 5, # 3, 7.

Assignment 9
Book Problems: Due Nov 10

Chapter 3, # 26; Chapter 4, # 9, 11, 15, 18.

Further Problem 5: Due Nov 7

Prove or disprove: *If* $E_1 \subseteq \mathbb{R}^n$ *and* $E_2 \subseteq \mathbb{R}^m$ *then* $|E_1 \times E_2|_e = |E_1|_e |E_2|_e$, *where the outer measures shown refer to* $\mathbb{R}^{n+m}$, $\mathbb{R}^n$ *and* $\mathbb{R}^m$ *respectively.*

Assignment 8
Book Problems: Due Nov 3

Chapter 3, # 17; Chapter 4, # 2, 3, 5, 7.

Assignment 7
Book Problems: Due Oct 27

Chapter 3, # 3, 19, 20, 21, 24.

Assignment 6
Book Problems: Due Oct 20

Chapter 3, # 3, 14, 15, 18, 25.

Further Problem 4: Due Oct 22

Prove (3.29). I expect lots of questions!

Assignment 5
Book Problems: Due Oct 15

Chapter 2, # 17; Chapter 3, # 9, 10, 12.

Assignment 4
Book Problems: Due Oct 6

Chapter 1, # 8(context: $\mathbb{R}^n$); Chapter 2, # 18; Chapter 3, # 5, 6, 7.

Assignment 3
Book Problems: Due Sept 29

Chapter 1, # 1k; Chapter 2, # 1, 6, 11, 14; the Exercise on p 1 of the Baby Riesz note.

Further Problem 3: Due Sept 24

Suppose that $\{f_n\}$ is a sequence of increasing real-valued functions defined on $\mathbb{R}$ that is *uniformly bounded,* i.e., there exist real numbers m, M such that $m \leq f_n(x) \leq M$ for all $x \in \mathbb{R}$. Prove that there exists a subsequence $f_{n_k}(x)$ that converges at every $x \in R$ (i.e., converges *pointwise*) to a bounded increasing function $f(x)$.
Suggested approach: enumerate the rationals and find a nested sequence of subsequences that converges at each rational number r. Here you may be guided by, and go beyond, Cantor's diagonal argument. Then define $f(x)$ to be the supremum of $f(r)$ for every $r \leq x$. Show that if f is continuous at x, then $f_{n_k}(x) \to f(x)$. Finally, you will most likely need to construct yet another subsequence of this one to take care of the countably many points of discontinuity of f.
This Theorem applies to functions defined on an interval as well, for we can extend the functions to $\mathbb{R}$ as constants outside the interval. It is useful to write subsequence indices as compositions of strictly increasing functions $s_j : \mathbb{N} \to \mathbb{N}$ rather than as multiply subscripted indices, for the chains of subscripts will become arbitrarily long. Note that the bounds on $f(x)$ are at worst those on the original sequence.

Assignment 2
Book Problems: Due Sept 22

Chapter 1, # 1efh; Chapter 2, # 2, 4, 5.

Further Problem 2: Due Sept 19

(a) Suppose that $f : [a, b] \to \mathbb{R}$ is an increasing function defined on the bounded closed interval $[a, b]$. Prove that $\lim_{x \uparrow x_o} f(x)$ exists at every $x_o \in (a, b]$, that $\lim_{x \downarrow x_o} f(x)$ exists at every $x_o \in [a, b)$ that f has at most countably many discontinuities, each of which is a jump discontinuity.
(b) Extend this result to monotone f defined on arbitrary intervals.

Assignment 1
Book Problems: Due Sept 15

Chapter 1, # 1adgj, 3.

Further Problem 1: Due Sept 12

Prove that the union of an arbitrary subcollection $\mathcal{C}$ of dyadic cubes is closed if the cubes in $\mathcal{C}$ each have edge lengths at least $\delta > 0$. See page 8, line -15ff, for the necessary terms.