Math 8602, Spring 2004 Assignments

Assignments are due at the start of class on the due date. Further Problems may be scored competitively! Neatness, legibility and cogency count! Give credit for help received, including books and hints from me and others; mention discussions. If a Further Problem is difficult, please include a "narrative" that tells what you went thru to reach your results. Further Problems should be written on standard size paper, and spiral-bound paper must be trimmed! There should be one-inch margins all around (name and problem number in the top margin are OK). Lined paper is fine, and using both sides is fine too. But if your handwriting fills up lines, double-space!

Assignment 12
Book Problems: Due May 5

Chapter 10, # 9, 12, 14, 17; Chapter 11, #2.

Further Problem 4: Due May 3

For an additive set function, consider $|V|(E) := \sup_{A\subseteq E} |\varphi(A)|$. "Do the math!" In this problem, elegance counts!

Assignment 11
Book Problems: Due Apr 30

Chapter 9, # 13; Chapter 10, # 5, 6.

Assignment 10
Book Problems: Due Apr 21

Chapter 9, # 9, 10, 12; Chapter 10, # 2, 3.

Further Problem 3: Due Apr 16

Prove another result of Schur: *If* K *is measurable on* $X \times Y$, $\int_X |K(s,t)|\,ds \le A$, $\int_Y |K(s,t)|\,dt \le B$ *and* $1 < p < \infty$ *then when* $f \in L^p(Y)$ *and* $Kf(s) := \int_Y K(s,t)f(t)$ *we have* $\|Kf\|_{p,ds} \le A^{1/p}B^{1/q}\|f\|_{p,dt}$. Include consideration of the existence of the integrals involved. Deal with the cases $p = 1$ and $p = \infty$ as well.

Assignment 9
Book Problems: Due Apr 14

Chapter 9, # 3, 4, 5, 7, 8.

Assignment 8
Book Problems: Due Mar 31

Chapter 9, # 1, 2; **Inner product spaces, and Hilbert spaces, v2: 33** and **35**; and, in addition:
Given that, for continuous f and g, $\langle f, g\rangle = \int_{-1}^{1} f(x)\overline{g(x)}\,\frac{dx}{\sqrt{1-x^2}}$, find the first four terms in the Gram-Schmidt orthogonalization of the sequence $\{x^n\}_{n=0}^{\infty}$.

Further Problem 2: Due Mar 22

Suppose that $T : X \to Y$, where X and Y are normed spaces and T is linear.
Let $S_T := \{C \in \mathbb{R} : \|Tx\|_Y \le C\|x\|_X$ for all $x \in X\}$ (S_T may be empty!).

Prove that $\|T\| := \inf S_T = \sup_{\|x\|_X \le 1} \|Tx\|_Y$, and that $\|T\| \in S_T$ if it is finite.

Let $X = Y$ be the collection of all sequences of real numbers that are eventually zero, both having the ℓ^2 norm. Find an example of a linear $T : X \to Y$ such that $S_T = \emptyset$.

Assignment 7
Book Problems: Due Mar 24

Chapter 8, # 10, 20, 21, 22, Prove that a finite-dimensional subspace of a normed space is closed (cf # 19).

Assignment 6
Book Problems: Due Mar 10

Chapter 8, # 13 – 17.

Assignment 5
Book Problems: Due Mar 3

Chapter 8, # 2, 6, 7, 11, 12.

Assignment 4
Book Problems: Due Feb 23

Chapter 8, # 1, 4, 5, 8.

Assignment 3
Book Problems: Due Feb 16

Chapter 3, # 27; Chapter 5, # 16; Chapter 7, # 12, 14, 15.

Assignment 2
Book Problems: Due Feb 9

Chapter 6, # 1b, 3, 4, 6, 10.

Further Problem 1: Due Feb 4

Let $f : \mathbb{R}^+ \to \mathbb{R}^+$ be increasing and suppose that $f(x)/x$ is decreasing on $\mathbb{R}^+$. Prove that $f(0)$ can be defined in such a way that f is absolutely continuous on $[0, \infty)$.

Assignment 1
Book Problems: Due Feb 2

Chapter 7, # 5, 8, 9, 10; Chapter 6, # 1a.