

Math 5467, Spring 2000: Sample Questions: Test 2

What are the formulas for the Fourier transforms of $f(t-h)$ and $f(\lambda t)$, $\lambda > 0$, in terms of $\hat{f}(\xi)$?

What are the formulas for the Fourier transforms of $f(-t)$ and $\overline{f(t)}$, in terms of $\hat{f}(\xi)$?

State Plancherel's Formula.

What is the Scaling Equation? What is the Fourier version of it?

What is the low-pass filter? What equation does the square of its absolute value satisfy?

Outline the proof that, if $g(t) \in L^2$ and if $\{g(t-n) : n \in \mathbb{Z}\}$ is an o.n. set, then $\sum_k |\hat{g}(\xi + 2\pi k)|^2 = 1$ a.e.

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Questions that start with a MRA being given

Given a MRA, how do we find the numbers $h(n)$?

Why, given a MRA, do we have (assuming $\sum |h(n)| < \infty$),

$$\sum h(n) = \sqrt{2}?$$

$$\sum (-1)^n h(n) = 0?$$

$$\sum (-1)^n h(n) \overline{h(n+2k)} = \delta_{k,0}?$$

What is the Cascade Formula?

Derive the Fourier version of the Scaling Equation from the Cascade Formula.

Given a MRA whose $\varphi(t)$ has compact support, what can be said about the $h(n)$'s?

Questions that start with a finite set of h_n 's being given

What condition on the low-pass filter did we use to prove that $\{\varphi(t-n) : n \in \mathbb{Z}\}$ is an o.n. set?

What other conditions did we assume about the h_n 's?

Why can we say $|m_o(\xi)| \leq 1$?

How did we show that

$$z_1 z_2 \cdots z_L - 1 = (z_1 - 1) z_2 \cdots z_L + (z_2 - 1) z_3 \cdots z_L + \cdots + (z_L - 1)$$

How did this help us show convergence in the Cascade Formula?

How did we show that the Cascade product is in L^2 ? What information did we gain about $\frac{1}{2\pi} \|\widehat{\Phi}\|_2^2$?

We expressed $p_N(\xi) = \prod_{k=1}^N m_o(\xi/2^k)$ as a sum of complex exponentials. How? What did we find out about the frequencies that appeared in that sum? How did this help us show that $\varphi(t)$ has compact support?

What did we find out about $\hat{\varphi}(0)$?

How did we show that $\sum_k |\hat{\varphi}(\xi + 2\pi k)|^2 = 1$ a.e.?

If you were to find a finite set of h_n 's that satisfies the conditions (h1)-(h5) would you be sure that they determine a MRA? Why?

If you had a *candidate* for a scaling function φ , what would you have to show in order to verify that it determined a MRA?

A wavelet question: What is the formula for ψ , in terms of φ ? What about $\widehat{\psi}$?