Math 5467, Spring 2000: Summary before Test 1

The properties of the spaces V_j in general, and the special case involving the V_j's determined by the box functions.

The (defining) properties of inner products

The Schwarz inequality

The parallelogram identity

The polarization formula

The idea of what a closed subspace is, and its definition

The idea of what a Hilbert space is, and the definition

The orthonormality of the Haar functions

The test for when $\operatorname{span} S$ is dense in a Hilbert space

The definition of orthonormalbasis

Be able to calculate the coefficients of some given function with respect to Haar functions

Be able to calculate the coefficients of some given periodic function with respect to the functions $e^{in\theta}$

Be able to find the projection of a given function onto one of the box-function V_j spaces

The formula for $P_X y$, the orthogonal (i.e. perpendicular) projection of y onto the closed subspace X.

The (Orthogonal) Projection Theorem

The Fischer-Riesz Theorem

Bessel's Inequality

Parseval's Formula and the Plancherel Theorem
(both are about orthonormalbases)

Things to take for granted:

Each V_j is a closed subspace of L^2, and thus is a Hilbert space in its own right.

$L^2(\mathbb{R})$ is a Hilbert space

$\{H_{jk}\}_{j\in\mathbb{Z},\ k\in\mathbb{Z}}$ is an orthonormal basis for $L^2(\mathbb{R})$

The union of all the V_j is dense in $L^2(\mathbb{R})$